Math 121 – Section 7.2 Solutions

9. $\cos\left(\sin^{-1}\dfrac{\sqrt{2}}{2}\right) = \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$

13. $\sec\left(\cos^{-1}\dfrac{1}{2}\right) = \sec\dfrac{\pi}{3} = 2$

19. $\sec\left[\sin^{-1}\left(-\dfrac{1}{2}\right)\right] = \sec\left(-\dfrac{\pi}{6}\right) = \dfrac{2}{\sqrt{3}}$

23. $\sin^{-1}\left[\sin\left(-\dfrac{7\pi}{6}\right)\right] = \sin^{-1}\dfrac{1}{2} = \dfrac{\pi}{6}$

31. $\sin[\tan^{-1}(-3)]$

    Let $\theta = \tan^{-1}(-3)$. Then $\tan\theta = -3$, where $-\dfrac{\pi}{2} < \theta < 0$, and:

$$\begin{aligned}
\tan^2\theta + 1 &= \sec^2\theta \\
(-3)^2 + 1 &= \sec^2\theta \\
\sec^2\theta &= 10 \\
\sec\theta &= \sqrt{10} \\
\cos\theta &= \frac{1}{\sqrt{10}}
\end{aligned}$$

    We took the positive root because θ lies in Quadrant IV. Then:

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin^2\theta + \left(\frac{1}{\sqrt{10}}\right)^2 &= 1 \\
\sin^2\theta + \frac{1}{10} &= 1 \\
\sin^2\theta &= \frac{9}{10} \\
\sin\theta &= -\frac{3}{\sqrt{10}}
\end{aligned}$$

    Therefore,

$$\sin[\tan^{-1}(-3)] = -\frac{3}{\sqrt{10}}$$

57. $\cos(\tan^{-1} u)$

Let $\theta = \tan^{-1} u$. Then $\tan\theta = u$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, and:

$$\begin{aligned}
\tan^2\theta + 1 &= \sec^2\theta \\
u^2 + 1 &= \sec^2\theta \\
\sec^2\theta &= u^2 + 1 \\
\sec\theta &= \sqrt{u^2+1} \\
\cos\theta &= \frac{1}{\sqrt{u^2+1}}
\end{aligned}$$

We took the positive root because θ lies in either Quadrant I or IV. Therefore,

$$\cos(\tan^{-1} u) = \frac{1}{\sqrt{u^2+1}}$$

59. $\tan(\sin^{-1} u)$

Let $\theta = \sin^{-1} u$. Then $\sin\theta = u$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, and:

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
u^2 + \cos^2\theta &= 1 \\
\cos^2\theta &= 1 - u^2 \\
\cos\theta &= \sqrt{1-u^2}
\end{aligned}$$

We took the positive root because θ lies in either Quadrant I or IV. Then:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{u}{\sqrt{1-u^2}}$$

Therefore,

$$\tan(\sin^{-1} u) = \frac{u}{\sqrt{1-u^2}}$$

61. $\sin(\sec^{-1} u)$

Let $\theta = \sec^{-1} u$. Then $\sec\theta = u$, where $0 \le \theta \le \pi$, and:

$$\cos\theta = \frac{1}{u}$$

Then:

$$\begin{aligned}
\sin^2\theta + \cos^2\theta &= 1 \\
\sin^2\theta + \left(\frac{1}{u}\right)^2 &= 1 \\
\sin^2\theta + \frac{1}{u^2} &= 1 \\
\sin^2\theta &= 1 - \frac{1}{u^2} \\
\sin^2\theta &= \frac{u^2-1}{u^2} \\
\sin\theta &= \frac{\sqrt{u^2-1}}{|u|}
\end{aligned}$$

where we put the absolute value on u since $\sin\theta \geq 0$. Therefore,

$$\sin(\sec^{-1} u) = \frac{\sqrt{u^2 - 1}}{|u|}$$