Math 121 – Section 7.3 Solutions

19. Establish $\csc\theta\cos\theta = \cot\theta$.

$$\begin{aligned}\csc\theta\cos\theta &= \frac{1}{\sin\theta}\cdot\cos\theta\\ &= \frac{\cos\theta}{\sin\theta}\\ &= \cot\theta\end{aligned}$$

23. Establish $\cos\theta(\tan\theta+\cot\theta) = \csc\theta$.

$$\begin{aligned}\cos\theta(\tan\theta+\cot\theta) &= \cos\theta\left(\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right)\\ &= \sin\theta+\frac{\cos^2\theta}{\sin\theta}\\ &= \frac{\sin^2\theta+\cos^2\theta}{\sin\theta}\\ &= \frac{1}{\sin\theta}\\ &= \csc\theta\end{aligned}$$

37. Establish $\sec u - \tan u = \dfrac{\cos u}{1+\sin u}$.

$$\begin{aligned}\sec u-\tan u &= \frac{1}{\cos u}-\frac{\sin u}{\cos u}\\ &= \frac{1-\sin u}{\cos u}\\ &= \frac{(1-\sin u)(1+\sin u)}{\cos u(1+\sin u)}\\ &= \frac{1-\sin^2 u}{\cos u(1+\sin u)}\\ &= \frac{\cos^2 u}{\cos u(1+\sin u)}\\ &= \frac{\cos u}{1+\sin u}\end{aligned}$$

41. Establish $1-\dfrac{\cos^2\theta}{1+\sin\theta} = \sin\theta$.

$$\begin{aligned}1-\frac{\cos^2\theta}{1+\sin\theta} &= \frac{1+\sin\theta-\cos^2\theta}{1+\sin\theta}\\ &= \frac{1+\sin\theta-(1-\sin^2\theta)}{1+\sin\theta}\\ &= \frac{\sin\theta+\sin^2\theta}{1+\sin\theta}\\ &= \frac{\sin\theta(1+\sin\theta)}{1+\sin\theta}\\ &= \sin\theta\end{aligned}$$

50. Establish $\dfrac{\cos v}{1+\sin v}+\dfrac{1+\sin v}{\cos v}=2\sec v$.

$$\begin{aligned}\frac{\cos v}{1+\sin v}+\frac{1+\sin v}{\cos v} &= \frac{\cos^2 v+(1+\sin v)^2}{\cos v(1+\sin v)}\\ &= \frac{\cos^2 v+1+2\sin v+\sin^2 v}{\cos v(1+\sin v)}\\ &= \frac{2+2\sin v}{\cos v(1+\sin v)}\\ &= \frac{2(1+\sin v)}{\cos v(1+\sin v)}\\ &= \frac{2}{\cos v}\\ &= 2\sec v\end{aligned}$$

70. Establish $\dfrac{\sin^2\theta-\tan\theta}{\cos^2\theta-\cot\theta}=\tan^2\theta$.

$$\begin{aligned}\frac{\sin^2\theta-\tan\theta}{\cos^2\theta-\cot\theta} &= \frac{\sin^2\theta-\frac{\sin\theta}{\cos\theta}}{\cos^2\theta-\frac{\cos\theta}{\sin\theta}}\\ &= \frac{\sin^3\theta\cos\theta-\sin^2\theta}{\sin^2\theta\cos^3\theta-\cos^2\theta}\\ &= \frac{\sin^2\theta(\sin\theta\cos\theta-1)}{\cos^2\theta(\sin\theta\cos\theta-1)}\\ &= \tan^2\theta\end{aligned}$$

91. Establish $\dfrac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta}=\tan\alpha\tan\beta$.

$$\begin{aligned}\frac{\tan\alpha+\tan\beta}{\cot\alpha+\cot\beta} &= \frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{\frac{\cos\alpha}{\sin\alpha}+\frac{\cos\beta}{\sin\beta}}\\ &= \frac{\sin^2\alpha\sin\beta\cos\beta+\sin\alpha\sin^2\beta\cos\alpha}{\sin\beta\cos^2\alpha\cos\beta+\sin\alpha\cos\alpha\cos^2\beta}\\ &= \frac{\sin\alpha\sin\beta(\sin\alpha\cos\beta+\sin\beta\cos\alpha)}{\cos\alpha\cos\beta(\sin\beta\cos\alpha+\sin\alpha\cos\beta)}\\ &= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta}\\ &= \tan\alpha\tan\beta\end{aligned}$$