Math 121 – Section 7.7 Solutions

7. Solve $2\sin\theta + 3 = 2$ on the interval $[0, 2\pi)$.

$$\begin{aligned} 2\sin\theta + 3 &= 2 \\ 2\sin\theta &= -1 \\ \sin\theta &= -\frac{1}{2} \end{aligned}$$

Therefore, the solutions are $\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$.

9. Solve $4\cos^2\theta = 1$ on the interval $[0, 2\pi)$.

$$\begin{aligned} 4\cos^2\theta &= 1 \\ \cos^2\theta &= \frac{1}{4} \\ \cos\theta &= \pm\frac{1}{2} \end{aligned}$$

Therefore, the solutions are $\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$.

15. Solve $\cos(2\theta) = -\frac{1}{2}$ on the interval $[0, 2\pi)$.

The solutions are:

$$\begin{aligned} 2\theta &= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{2\pi}{3} + 2\pi, \frac{4\pi}{3} + 2\pi \\ 2\theta &= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3} \end{aligned}$$

$$\boxed{\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

24. Solve $5\csc\theta - 3 = 2$ on the interval $[0, 2\pi)$.

$$\begin{aligned} 5\csc\theta - 3 &= 2 \\ 5\csc\theta &= 5 \\ \csc\theta &= 1 \\ \sin\theta &= 1 \end{aligned}$$

Therefore, the solution is $\boxed{\theta = \frac{\pi}{2}}$.

31. Solve $\sin\theta = \dfrac{1}{2}$ and give a general formula for all solutions. Then list six solutions.

The two base solutions are $\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$.

The remaining solutions are these solutions plus or minus a multiple of 2π:

$$\boxed{\theta = \frac{\pi}{6} \pm 2n\pi, \quad \frac{5\pi}{6} \pm 2n\pi \quad \text{where} \quad n = 0, 1, 2, \ldots}$$

Six solutions are:

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$$

37. Solve $\cos(2\theta) = -\dfrac{1}{2}$ and give a general formula for all solutions. Then list six solutions.

Two base solutions are:

$$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$$
$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

The remaining solutions are these solutions plus or minus a multiple of π (not 2π since the original equation has 2θ):

$$\boxed{\theta = \frac{\pi}{3} \pm n\pi, \quad \frac{2\pi}{3} \pm n\pi \quad \text{where} \quad n = 0, 1, 2, \ldots}$$

Six solutions are:

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$$

41. One solution to $\sin\theta = 0.4$ on the interval $[0, 2\pi)$ is:

$$\theta = \sin^{-1} 0.4 \approx \boxed{0.41}$$

The other solution is:

$$\theta = \pi - \sin^{-1} 0.4 \approx \boxed{2.73}$$