Math 121 – Section 9.1 Solutions

39. The point $(r, \theta) = \left(3, \frac{\pi}{2}\right)$ in rectangular coordinates is:

$$x = r\cos\theta = 3\cos\frac{\pi}{2} = 0$$
$$y = r\sin\theta = 3\sin\frac{\pi}{2} = 3$$

So $\boxed{(x, y) = (0, 3)}$.

41. The point $(r, \theta) = (-2, 0)$ in rectangular coordinates is:

$$x = r\cos\theta = -2\cos 0 = -2$$
$$y = r\sin\theta = -2\sin 0 = 0$$

So $\boxed{(x, y) = (-2, 0)}$.

43. The point $(r, \theta) = (6, 150^\circ)$ in rectangular coordinates is:

$$x = r\cos\theta = 6\cos 150^\circ = 6\left(-\frac{\sqrt{3}}{2}\right) = -3\sqrt{3}$$
$$y = r\sin\theta = 6\sin 150^\circ = 6\left(\frac{1}{2}\right) = 3$$

So $\boxed{(x, y) = (-3\sqrt{3}, 3)}$.

55. The point $(x, y) = (3, 0)$ in polar coordinates is:

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 0^2} = 3$$
$$\cos\theta = \frac{x}{r} = \frac{3}{3} = 1$$
$$\sin\theta = \frac{y}{r} = \frac{0}{3} = 0$$

So $\boxed{(r, \theta) = (3, 0)}$.

59. The point $(x, y) = (1, -1)$ in polar coordinates is:

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$
$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$
$$\sin\theta = \frac{y}{r} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

So $\boxed{(r, \theta) = \left(\sqrt{2}, \frac{7\pi}{4}\right)}$.

62. The point $(x, y) = \left(-2, -2\sqrt{3}\right)$ in polar coordinates is:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{4 + 12} = 4 \\ \cos\theta &= \frac{x}{r} = -\frac{2}{4} = -\frac{1}{2} \\ \sin\theta &= \frac{y}{r} = -\frac{2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2} \end{aligned}$$

So $\boxed{(r, \theta) = \left(4, \frac{4\pi}{3}\right)}$.

67. The equation $2x^2 + 2y^2 = 3$ in polar coordinates is:

$$\begin{aligned} 2x^2 + 2y^2 &= 3 \\ 2(x^2 + y^2) &= 3 \\ x^2 + y^2 &= \frac{3}{2} \\ r^2 &= \frac{3}{2} \\ r &= \sqrt{\frac{3}{2}} \end{aligned}$$

75. The equation $r = \cos\theta$ in rectangular coordinates is:

$$\begin{aligned} r &= \cos\theta \\ \sqrt{x^2 + y^2} &= \frac{x}{r} \\ \sqrt{x^2 + y^2} &= \frac{x}{\sqrt{x^2 + y^2}} \\ x^2 + y^2 &= x \\ x^2 - x + y^2 &= 0 \\ \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + y^2 &= 0 \\ \left(x - \frac{1}{2}\right)^2 + y^2 &= \frac{1}{4} \end{aligned}$$