

## MATH 121 Dias Exam 3, Spring 2008 Answers

1. (a)  $1200\pi$  rad/min  
 (b) i.  $2160\pi$  ft/min  
 ii.  $\frac{270\pi}{11}$  mph
2. (a)  $\sin t$ : domain is all real numbers; range is  $[-1, 1]$ ; period is  $2\pi$   
 $\tan t$ : domain is all  $t$  except  $t = \pm\frac{\pi}{2} \pm 2n\pi$  where  $n = 0, 1, 2, \dots$ ; range is all real numbers; period is  $\pi$   
 (b) i.  $\frac{\sqrt{3}}{2}$   
 ii.  $\frac{\sqrt{3}}{2}$   
 iii.  $\sqrt{3}$
3. (a)  $\cos t = -\frac{\sqrt{21}}{5}$   
 (b)  $\tan t = \frac{2}{\sqrt{21}}$ ,  $\csc(-t) = -\csc t = \frac{5}{2}$ ,  $\sec(-t) = \sec(t) = -\frac{5}{\sqrt{21}}$
4.  $A = 7$ ,  $b = 16$ ,  $c = -8\pi$
5. (a)  $\sin(\pi - x) = \sin \pi \cos x - \cos \pi \sin x = \sin x$   
 (b)

$$\begin{aligned}
 \frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x} &= \frac{(\cos x)^2 + (1 + \sin x)^2}{\cos x(1 + \sin x)} \\
 &= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1 + \sin x)} \\
 &= \frac{2 + 2\sin x}{\cos x(1 + \sin x)} \\
 &= \frac{2(1 + \sin x)}{\cos x(1 + \sin x)} \\
 &= \frac{2}{\cos x} \\
 &= 2 \sec x
 \end{aligned}$$

6. (a)  $\cos 2x = \cos(x + x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$   
 (b)

$$\begin{aligned}
 1 + \cos 2x &= 1 + \cos^2 x - \sin^2 x \\
 &= 1 + \cos^2 x - (1 - \cos^2 x) \\
 &= 2 \cos^2 x
 \end{aligned}$$

(c)

$$\begin{aligned}
 1 + \cos 2x &= 2 \cos^2 x \\
 2 \cos^2 x &= 1 + \cos 2x \\
 \cos^2 x &= \frac{1 + \cos 2x}{2} \\
 \cos x &= \pm \sqrt{\frac{1 + \cos 2x}{2}}
 \end{aligned}$$

$$(d) \cos \frac{\pi}{8} = \sqrt{\frac{1 + \cos \frac{\pi}{4}}{2}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

7. (a)  $\sin^{-1} x$ : domain is  $[-1, 1]$ ; range is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$   
 $\cos^{-1} x$ : domain is  $[-1, 1]$ ; range is  $[0, \pi]$   
 $\tan^{-1} x$ : domain is all real numbers; range is  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- (b)  $x = \pm \frac{\pi}{2} \pm n\pi$ ,  $x = \frac{\pi}{4} \pm n\pi$  where  $n = 0, 1, 2, \dots$