Math 310 Homework 13 Solutions

Chapter 6, Section 4

1. (a) $\mathbf{z} = \begin{bmatrix} 4+2i \\ 4i \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} -2 \\ 2+i \end{bmatrix}$

   i. $||\mathbf{z}|| = \sqrt{\mathbf{z}^H\mathbf{z}} = \sqrt{|4+2i|^2 + |4i|^2} = \sqrt{4^2+2^2+4^2} = 6$

   ii. $||\mathbf{w}|| = \sqrt{\mathbf{w}^H\mathbf{w}} = \sqrt{|-2|^2 + |2+i|^2} = \sqrt{(-2)^2+2^2+1^2} = 3$

   iii. $\langle \mathbf{z}, \mathbf{w} \rangle = \mathbf{w}^H\mathbf{z} = \begin{bmatrix} -2 & 2-i \end{bmatrix} \begin{bmatrix} 4+2i \\ 4i \end{bmatrix} = (-2)(4+2i) + (2-i)(4i) = -4+4i$

   iv. $\langle \mathbf{w}, \mathbf{z} \rangle = \overline{\langle \mathbf{z}, \mathbf{w} \rangle} = -4-4i$

4. (a) not Hermitian (one of the diagonal entries is complex); not normal since $MM^H \neq M^HM$

   (c) not Hermitian (real but not symmetric); normal since $MM^H = M^HM$

   (e) not Hermitian ($M \neq M^H$); normal since $MM^H = M^HM$

5. (a) The eigenvalues and eigenvectors of the matrix are:

$$\lambda = 1 : \begin{bmatrix} -1 \\ 1 \end{bmatrix}; \ \lambda = 3 : \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

   The eigenvectors are orthogonal. To make them unit vectors we divide by their norms. The norm of each vector is $\sqrt{2}$. Therefore, the unitary diagonalizing matrix and the diagonal matrix are:

$$U = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \ D = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

   (e) The eigenvalues and eigenvectors of the matrix are:

$$\lambda = 1 : \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}; \ \lambda = -1 : \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

   The eigenvectors are orthogonal. To make them unit vectors we divide by their norms. Therefore, the unitary diagonalizing matrix and the diagonal matrix are:

$$U = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \ D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$