Math 310 Homework 2 Solutions

Chapter 1, Section 3

2. (a) Yes. $\begin{bmatrix} 15 & 19 \\ 4 & 0 \end{bmatrix}$

   (b) No.

   (c) Yes. $\begin{bmatrix} 19 & 21 \\ 17 & 21 \\ 8 & 10 \end{bmatrix}$

   (d) Yes. $\begin{bmatrix} 36 & 10 & 56 \\ 10 & 3 & 16 \end{bmatrix}$

7b. $(AB)^T = B^T A^T = \begin{bmatrix} 5 & 15 & 0 \\ 14 & 42 & 16 \end{bmatrix}$

10. $A = A^2 = A^3 = \ldots = A^n = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

13a. $\mathbf{b} = 2\mathbf{a}_1 + \mathbf{a}_2$

13c. $\mathbf{c} = -\frac{5}{2}\mathbf{a}_1 - \frac{1}{4}\mathbf{a}_2$

14b. $\mathbf{b} = \mathbf{a}_1 + \mathbf{a}_2 \Rightarrow$ the system is consistent. In fact, $x_1 = 1$ and $x_2 = 1$.

14c. The system is inconsistent. No linear combination of the columns of A will produce the given $\mathbf{b}$.

24. Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ be an arbitrary 2×2 matrix where $a, b, c \in \mathbb{R}$ but not all zero. Then we have:

$$A^2 = AA = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & b^2 + c^2 \end{bmatrix}$$

In order for A^2 to be the zero matrix we must have:

$$\begin{aligned} a^2 + b^2 &= 0 \\ ab + bc &= 0 \\ b^2 + c^2 &= 0 \end{aligned}$$

However, in order for the first and third equation to be satisfied we must have $a = b = c = 0$. So, there exist no such a, b, c (not all zero) such that A^2 is the zero matrix.

26. (a) Since $A \in \mathbb{R}^{m \times n}$ we have $A^T \in \mathbb{R}^{n \times m}$. The product $A^T A$ can be performed since the number of columns of A^T equals the number of rows of A. In fact, $A^T A \in \mathbb{R}^{n \times n}$. The product AA^T can be performed since the number of columns of A equals the number of rows of A^T. In fact, $AA^T \in \mathbb{R}^{m \times m}$.

    (b) To show that $A^T A$ and AA^T are symmetric we need to show that $(A^T A)^T = A^T A$ and $(AA^T)^T = AA^T$, respectively. This is not too difficult:

$$\begin{aligned} (A^T A)^T &= A^T (A^T)^T = A^T A \\ (AA^T)^T &= (A^T)^T A^T = AA^T \end{aligned}$$

In the first step of both computations above, we used the fact that $(AB)^T = B^T A^T$.