Math 310 Homework 3 Solutions

Chapter 1, Section 4

3. (a) $E = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}$, (b) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, (c) $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

6. (a)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 6 & 4 & 5 \\ 4 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \quad \left\{ E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix} \quad \left\{ E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \right\}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \quad \left\{ E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \right\}$$

$$\boxed{E_3 E_2 E_1 A = U = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}}$$

(b)

$$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}, \; E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\boxed{L = E_1^{-1} E_2^{-1} E_3^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix}}$$

8. (a) $L = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

(b) $L = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix}$

(c) $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 4 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix}$

(d) $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix}$, $U = \begin{bmatrix} -2 & 1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$

10. (a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, (b) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$, (c) $\begin{bmatrix} -4 & 3 \\ \frac{3}{2} & -1 \end{bmatrix}$, (d) $\begin{bmatrix} \frac{1}{3} & 0 \\ -1 & \frac{1}{3} \end{bmatrix}$,

(e) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, (f) $\begin{bmatrix} 3 & 0 & -5 \\ 0 & \frac{1}{3} & 0 \\ -1 & 0 & 2 \end{bmatrix}$

15. From the given statement, $2\mathbf{a}_1 + \mathbf{a}_2 - 4\mathbf{a}_3 = \mathbf{0}$, one solution to $A\mathbf{x} = \mathbf{0}$ is $x_1 = 2$, $x_2 = 1$, $x_3 = -4$. We know that either this is the only solution or there are an infinite number of solutions. However, we know that $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ is also a solution. Therefore, since we have found more than one solution there must be an infinite number of solutions.

    If A were nonsingular, there would only be one solution: $\mathbf{x} = A^{-1}\mathbf{0} = \mathbf{0}$. However, there is more than one solution so A must be singular.

Chapter 2, Section 1

3. (a) 1, (b) 4, (c) 0, (d) 58

6.

$$\begin{aligned}
\begin{vmatrix} 2-\lambda & 4 \\ 3 & 3-\lambda \end{vmatrix} &= 0 \\
(2-\lambda)(3-\lambda) - (4)(3) &= 0 \\
6 - 5\lambda + \lambda^2 - 12 &= 0 \\
\lambda^2 - 5\lambda - 6 &= 0 \\
(\lambda - 6)(\lambda + 1) &= 0 \\
\lambda = 6,\ \lambda &= -1
\end{aligned}$$