Math 310 Homework 4 Solutions

Chapter 2, Section 1

4. (a) $\det A = 2$ (use 2×2 rule)
   (b) $\det A = -4$ (expand by cofactors of first row)
   (c) $\det A = 0$ (expand by cofactors of first row)
   (d) $\det A = 0$ (second column contains all zeros)

Chapter 2, Section 2

1. (a) $\det A = -24$ (can expand by cofactors of first row or first column)
   (b) $\det A = 30$ (first replace row 4 with its sum with row 1 to get [0 0 0 5] in row 4 - this does not change the determinant since this is a type III row operation; the resulting matrix is upper triangular so its determinant is the product of the diagonal entries)
   (c) $\det A = 1$ (expand by cofactors of row 1 - M_{14} is the identity matrix whose determinant is 1)

3. (a) $\det A = 0 \Rightarrow A$ is singular
   (b) $\det A = 2 \Rightarrow A$ is nonsingular
   (c) $\det A = -3 \Rightarrow A$ is nonsingular
   (d) $\det A = 2 \Rightarrow A$ is nonsingular
   (e) $\det A = 0 \Rightarrow A$ is singular
   (f) $\det A = 0 \Rightarrow A$ is singular

4. The determinant is:

$$\begin{aligned}\begin{vmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{vmatrix} &= 1 \cdot \begin{vmatrix} 9 & c \\ c & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & c \\ 1 & 3 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 9 \\ 1 & c \end{vmatrix} \\ &= (27 - c^2) - (3 - c) + (c - 9) \\ &= 15 + 2c - c^2\end{aligned}$$

The matrix is singular when the determinant is 0. Therefore,

$$\begin{aligned}15 + 2c - c^2 &= 0 \\ c^2 - 2c - 15 &= 0 \\ (c-5)(c+3) &= 0 \\ c = 5,&\ c = -3\end{aligned}$$

12. (a)

$$\begin{aligned}\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} \xrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_1} &\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 \\ 0 & x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix} \\ &= 1 \cdot \begin{vmatrix} x_2 - x_1 & x_2^2 - x_1^2 \\ x_3 - x_1 & x_3^2 - x_1^2 \end{vmatrix} \\ &= (x_2 - x_1)(x_3^2 - x_1^2) - (x_3 - x_1)(x_2^2 - x_1^2) \\ &= (x_2 - x_1)(x_3 - x_1)(x_3 + x_1) - (x_3 - x_1)(x_2 - x_1)(x_2 + x_1) \\ &= (x_2 - x_1)(x_3 - x_1)[(x_3 + x_1) - (x_2 + x_1)] \\ &= (x_2 - x_1)(x_3 - x_1)(x_3 - x_2)\end{aligned}$$

(b) The scalars x_1, x_2, and x_3 must be distinct for the matrix V to be nonsingular.

Chapter 2, Section 3

1. (a) i. $\det A = -7$

ii. $\text{adj } A = \begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}$

iii. $A^{-1} = \frac{1}{-7}\begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix}$

(c) i. $\det A = 3$

ii. $\text{adj } A = \begin{bmatrix} -3 & 5 & 2 \\ 0 & 1 & 1 \\ 6 & -8 & -5 \end{bmatrix}$

iii. $A^{-1} = \frac{1}{3}\begin{bmatrix} -3 & 5 & 2 \\ 0 & 1 & 1 \\ 6 & -8 & -5 \end{bmatrix}$

2. (a)

$$x_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \frac{-5}{-7} = \frac{5}{7}$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}}{-7} = \frac{-8}{-7} = \frac{8}{7}$$

(b)

$$x_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 2 & 3 \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix}} = \frac{-11}{-5} = \frac{11}{5}$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 2 & 2 \\ 3 & 5 \end{vmatrix}}{-5} = \frac{4}{-5} = -\frac{4}{5}$$

(c)

$$x_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 0 & 1 & -3 \\ 8 & 5 & 1 \\ 2 & -1 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -3 \\ 4 & 5 & 1 \\ -2 & -1 & 4 \end{vmatrix}} = \frac{24}{6} = 4$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 2 & 0 & -3 \\ 4 & 8 & 1 \\ -2 & 2 & 4 \end{vmatrix}}{6} = \frac{-12}{6} = -2$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{\begin{vmatrix} 2 & 1 & 0 \\ 4 & 5 & 8 \\ -2 & -1 & 2 \end{vmatrix}}{6} = \frac{12}{6} = 2$$