

Math 310 Quiz 1 Solution

Consider the following linear system of equations:

$$\begin{array}{rccccrcr} & & 4x_2 & + & 2x_3 & = & 1 \\ x_1 & + & 3x_2 & + & x_3 & = & 0 \\ 2x_1 & + & 2x_2 & & & = & 1 \end{array}$$

- (a) Write the augmented matrix for the system.
- (b) Put the augmented matrix into row echelon form.
- (c) Find all solutions to the system.

Solution:

- (a) The augmented matrix is:

$$\left[\begin{array}{ccc|c} 0 & 4 & 2 & 1 \\ 1 & 3 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array} \right]$$

- (b)

$$\begin{aligned} \left[\begin{array}{ccc|c} 0 & 4 & 2 & 1 \\ 1 & 3 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array} \right] & \xrightarrow[\begin{array}{l} R_1 \rightarrow R_2 \\ R_2 \rightarrow R_1 \end{array}]{\begin{array}{l} R_1 \rightarrow R_2 \\ R_2 \rightarrow R_1 \end{array}} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 4 & 2 & 1 \\ 2 & 2 & 0 & 1 \end{array} \right] \\ & \xrightarrow{R_3 \rightarrow R_3 + (-2R_1)} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 4 & 2 & 1 \\ 0 & -4 & -2 & 1 \end{array} \right] \\ & \xrightarrow{R_3 \rightarrow R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 4 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{array} \right] \\ & \xrightarrow{R_2 \rightarrow \frac{1}{4}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 0 & 2 \end{array} \right] \end{aligned}$$

- (c) The last row of the above matrix says that:

$$0x_1 + 0x_2 + 0x_3 = 2 \quad \text{or} \quad 0 = 2$$

However, this is clearly not true. So the system of equations is *inconsistent* and has *no solutions*.