Math 310 Quiz 2 Solution

1. In each case below, state whether or not the inverse exists. If it does, find it. If it doesn't, explain why not.

   (a) $A = \begin{bmatrix} 0 & 1 \\ 2 & -2 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

2. Find an elementary matrix E such that $EC = D$ for the following pair of matrices:

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 2 & -1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & -5 & -5 \end{bmatrix}$$

Solution:

1. (a) To find the inverse, we augment A with I and row reduce:

$$\left[\begin{array}{cc|cc} 0 & 1 & 1 & 0 \\ 2 & -2 & 0 & 1 \end{array}\right] \xrightarrow[R_2 \to R_1]{R_1 \to R_2} \left[\begin{array}{cc|cc} 2 & -2 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{array}\right]$$
$$\xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & -1 & 0 & \frac{1}{2} \\ 0 & 1 & 1 & 0 \end{array}\right]$$
$$\xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{cc|cc} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 1 & 0 \end{array}\right]$$

   Therefore, the inverse of A is:

$$A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$$

   (b) To find the inverse, we augment A with I and row reduce:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$$

   We cannot possibly row reduce A to the identity matrix because we have a row of zeros. Therefore, A is singular and its inverse doesn't exist.

2. D is obtained by replacing the third row of C with its sum with -2 times the first row. Therefore, the elementary matrix is obtained by performing the same operation on the identity matrix. The result is:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$