Math 310 Quiz 3 Solution

1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 2 & 3 & 2 \end{bmatrix}$.

   (a) Compute $\det A$.

   (b) Is A singular? Why or why not?

   Solution:

   (a) To compute the determinant we expand by cofactors of the second row since it has the most zeros:

$$\begin{aligned}
&\det A = 0 \cdot \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} - (-2) \cdot \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + 0 \cdot \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\
&\det A = 0 + 2[(1)(2) - (1)(2)] + 0 \\
&\boxed{\det A = 0}
\end{aligned}$$

   (b) Since $\det A = 0$, A is singular.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 0 & -4 & 1 \end{bmatrix}$. The LU-factorization of A is:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 11 \end{bmatrix}$$

   Compute $\det A$.

   Solution: We can use the following property of determinants:

$$\det AB = \det A \det B$$

   Applied to this problem we have:

$$\det A = \det LU = \det L \det U$$

   Since L and U are triangular, their determinants are the product of the entries on the main diagonal. Thus,

$$\begin{aligned}
&\det L = (1)(1)(1) = 1 \\
&\det U = (1)(2)(11) = 22 \\
\Rightarrow \quad &\det A = \det L \det U = (1)(22) = 22
\end{aligned}$$