Math 310 Quiz 4 Solution

1. Suppose $\det A = 2$ and

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{3}{2} \end{bmatrix}$$

What is adj A?

Solution: Using the definition $A^{-1} = \dfrac{1}{\det A}\text{adj } A$, we compute adj A as follows:

$$\text{adj } A = (\det A)A^{-1}$$

$$\text{adj } A = 2 \cdot \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{3}{2} \end{bmatrix}$$

$$\boxed{\text{adj } A = \begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}}$$

2. Use Cramer's rule to solve the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_2 + x_3 &= 0 \\ 2x_2 - 3x_3 &= 0 \end{aligned}$$

Solution: We write the system of equations in matrix-vector form:

$$A\mathbf{x} = \mathbf{b}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\det A = -5$. Using Cramer's Rule we find that:

$$x_1 = \frac{\det A_1}{\det A} = \frac{\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & -3 \end{vmatrix}}{-5} = \frac{-5}{-5} = 1$$

$$x_2 = \frac{\det A_2}{\det A} = \frac{\begin{vmatrix} 1 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{vmatrix}}{-5} = \frac{0}{-5} = 0$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{vmatrix}}{-5} = \frac{0}{-5} = 0$$