# Math 310 Quiz 6 Solution

Consider the following linear operator on $\mathbb{R}^2$:

$$L(\mathbf{x}) = \begin{bmatrix} x_1 + 2x_2 \\ x_1 - x_2 \end{bmatrix}$$

Find the matrix representation of $L$ with respect to each of the bases below:

1. $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
2. $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

**Solution**:

1. $L(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $L(\mathbf{e}_2) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$. Therefore, the matrix representation of $L$ with respect to the standard basis is:

$$\boxed{A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}}$$

2. $L(\mathbf{v}_1) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $L(\mathbf{v}_2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. However, we are not done yet. We must write these two vectors with respect to the $\mathbf{v}_1$, $\mathbf{v}_2$ basis vectors:

$$L(\mathbf{v}_1) = \begin{bmatrix} 3 \\ 0 \end{bmatrix} = 0\mathbf{v}_1 + 3\mathbf{v}_2$$
$$L(\mathbf{v}_2) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1\mathbf{v}_1 + 0\mathbf{v}_2$$

Therefore, the matrix representation of $L$ with respect to the $\mathbf{v}$ basis vectors is:

$$\boxed{A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}}$$