Math 180, Exam 1, Study Guide
Problem 1 Solution

1. What is the slope of the linear function $f(x)$ whose graph goes through the points $(1, 2)$ and $(4, 4)$? What is the value of $f(7)$?

Solution: The slope of the line that goes through the points $(1, 2)$ and $(4, 4)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{4-1} = \frac{2}{3}$$

Using the slope above and the point $(1, 2)$, we write an equation for the line as follows:

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= \frac{2}{3}(x-1) \\ y - 2 &= \frac{2}{3}x - \frac{2}{3} \\ y &= \frac{2}{3}x + \frac{4}{3} \end{aligned}$$

Therefore, the linear function is $f(x) = \frac{2}{3}x + \frac{4}{3}$. The value of $f(7)$ is:

$$f(7) = \frac{2}{3}(7) + \frac{4}{3} = 6$$

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Problem 2 Solution

2. Find $\displaystyle\lim_{x\to 2} \frac{x^2-3x+2}{x^2-x-2}$.

Solution: When substituting $x = 2$ into the function $f(x) = \dfrac{x^2-3x+2}{x^2-x-2}$ we find that

$$\frac{x^2-3x+2}{x^2-x-2} = \frac{2^2-3(2)+2}{2^2-2-2} = \frac{0}{0}$$

which is indeterminate. We can resolve the indeterminacy by factoring.

$$\begin{aligned}\lim_{x\to 2} \frac{x^2-3x+2}{x^2-x-2} &= \lim_{x\to 2} \frac{(x-1)(x-2)}{(x+1)(x-2)} \\ &= \lim_{x\to 2} \frac{x-1}{x+1} \\ &= \frac{2-1}{2+1} \\ &= \boxed{\frac{1}{3}}\end{aligned}$$

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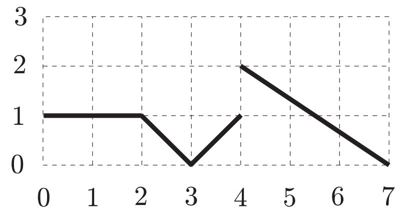
Problem 3 Solution

3. The graph of function $f(x)$ is below. At what point or points is $f(x)$ not continuous?



Solution: The function is not continuous at $x = 4$ because the one-sided limits

$$\lim_{x \to 4^-} f(x) = 1, \qquad \lim_{x \to 4^+} f(x) = 2$$

are not equal to each other.

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Problem 4 Solution

4. Find the derivatives of the following functions using the basic rules. Do not simplify your answer.

(a) $x^3 + x^{1/3}$, (b) $x^2 e^x$, (c) $\dfrac{2+x}{3+x^2}$

Solution:

(a) Use the Power Rule.

$$(x^3 + x^{1/3})' = \boxed{3x^2 + \frac{1}{3}x^{-2/3}}$$

(b) Use the Product Rule.

$$\begin{aligned}(x^2 e^x)' &= x^2(e^x)' + (x^2)'e^x \\ &= \boxed{x^2e^x + 2xe^x}\end{aligned}$$

(c) Use the Quotient Rule.

$$\begin{aligned}\left(\frac{2+x}{3+x^2}\right)' &= \frac{(3+x^2)(2+x)' - (2+x)(3+x^2)'}{(3+x^2)^2} \\ &= \boxed{\frac{(3+x^2) - (2+x)(2x)}{(3+x^2)^2}}\end{aligned}$$

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Problem 5 Solution

5. Find the derivatives of the following functions using the basic rules. Do not simplify your answer.

(a) $1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$, (b) $(x-1)e^x$, (c) $\frac{1}{\sqrt{x}-1}$

Solution:

(a) Use the Power Rule.

$$\left(1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3\right)' = \boxed{1 + x + \frac{1}{2}x^2}$$

(b) Use the Product Rule.

$$\begin{aligned}[(x-1)e^x]' &= (x-1)(e^x)' + (x-1)'e^x \\ &= \boxed{(x-1)e^x + e^x}\end{aligned}$$

(c) Use the Quotient Rule.

$$\begin{aligned}\left(\frac{1}{\sqrt{x}-1}\right)' &= \frac{(\sqrt{x}-1)(1)' - (1)(\sqrt{x}-1)'}{(\sqrt{x}-1)^2} \\ &= \boxed{\frac{0 - \frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^2}}\end{aligned}$$

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Problem 6 Solution

6. Find the equation of the tangent line to $y = f(x)$ at $x = 2$ for the function $f(x) = -x^2 + 7x$. Do not simplify your answer.

Solution: The derivative $f'(x)$ is found using the Power Rule.

$$f'(x) = (-x^2 + 7x)' = -2x + 7$$

At $x = 2$ the values of f and f' are:

$$f(2) = -2^2 + 7(2) = 10$$
$$f'(2) = -2(2) + 7 = 3$$

We now know that the point $(2, 10)$ is on the tangent line and that the slope of the tangent line is 3. Therefore, an equation for the tangent line in point-slope form is:

$$\boxed{y - 10 = 3(x - 2)}$$

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Problem 7 Solution

7. Let $f(x) = x^2 + 3$.

(a) Find the average rate of change of $f(x)$ over the interval $1 \leq x \leq 3$.

(b) Find the instantaneous rate of change of $f(x)$ at $x = 2$.

Solution:

(a) The average rate of change formula is:

$$\text{average ROC} = \frac{f(b) - f(a)}{b - a}$$

Using $f(x) = x^2 + 3$, $b = 3$, and $a = 1$ we have:

$$\text{average ROC} = \frac{(3^2 + 3) - (1^2 + 3)}{3 - 1} = \boxed{4}$$

(b) The instantaneous rate of change at $x = 2$ is $f'(2)$. The derivative $f'(x)$ is:

$$f'(x) = 2x$$

At $x = 2$ we have:

$$\text{instantaneous ROC} = f'(2) = 2(2) = \boxed{4}$$

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Problem 8 Solution

8. Let $f(x) = x^2$. Use the definition of the derivative as the limit of the difference quotient to find $f'(3)$.

Solution: There are two possible difference quotients we can use to evaluate $f'(3)$. One is:

$$f'(3) = \lim_{h \to 0} \frac{f(h+3) - f(3)}{h} = \lim_{h \to 0} \frac{(h+3)^2 - 3^2}{h}.$$

The other is:

$$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{x^2 - 3^2}{x - 3}$$

Evaluating the first limit above we have:

$$\begin{aligned}
f'(3) &= \lim_{h \to 0} \frac{(h+3)^2 - 3^2}{h} \\
&= \lim_{h \to 0} \frac{h^2 + 6h + 9 - 9}{h} \\
&= \lim_{h \to 0} \frac{h^2 + 6h}{h} \\
&= \lim_{h \to 0} (h + 6) \\
&= 0 + 6 \\
&= \boxed{6}
\end{aligned}$$

Evaluating the second limit we have:

$$\begin{aligned}
f'(3) &= \lim_{x \to 3} \frac{x^2 - 3^2}{x - 3} \\
&= \lim_{x \to 3} \frac{x^2 - 9}{x - 3} \\
&= \lim_{x \to 3} \frac{(x+3)(x-3)}{x - 3} \\
&= \lim_{x \to 3} (x + 3) \\
&= 3 + 3 \\
&= \boxed{6}
\end{aligned}$$

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Problem 9 Solution

9. Use the table below, which shows values of $f(x)$ for x near 2,

x	1.8	1.9	2.0	2.1	2.2
$f(x)$	2.24	2.27	2.30	2.33	2.37

to find the slope of a secant line that is an estimate for $f'(2)$.

Solution: An approximate value for $f'(2)$ is

$$f'(2) \approx \frac{f(2.1) - f(2.0)}{2.1 - 2.0} = \frac{2.33 - 2.30}{0.1} = \boxed{0.3}$$

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Problem 10 Solution

10. Find $\lim_{x\to 1} \dfrac{\sqrt{3x+1}-\sqrt{2x+2}}{x-1}$.

Solution: When substituting $x = 1$ into the function $f(x) = \dfrac{\sqrt{3x+1}-\sqrt{2x+1}}{x-1}$, we find that

$$\frac{\sqrt{3x+1}-\sqrt{2x+2}}{x-1} = \frac{\sqrt{3(1)+1}-\sqrt{2(1)+2}}{1-1} = \frac{0}{0}$$

which is indeterminate. We can resolve the indeterminacy by multiplying $f(x)$ by the "conjugate" of the numerator divided by itself.

$$\begin{aligned}
\lim_{x\to 1} \frac{\sqrt{3x+1}-\sqrt{2x+2}}{x-1} &= \lim_{x\to 1} \frac{\sqrt{3x+1}-\sqrt{2x+1}}{x-1} \cdot \frac{\sqrt{3x+1}+\sqrt{2x+2}}{\sqrt{3x+1}+\sqrt{2x+2}} \\
&= \lim_{x\to 1} \frac{(3x+1)-(2x+2)}{(x-1)(\sqrt{3x+1}+\sqrt{2x+2})} \\
&= \lim_{x\to 1} \frac{(x-1)}{(x-1)(\sqrt{3x+1}+\sqrt{2x+2})} \\
&= \lim_{x\to 1} \frac{1}{\sqrt{3x+1}+\sqrt{2x+2}} \\
&= \frac{1}{\sqrt{3(1)+1}+\sqrt{2(2)+2}} \\
&= \boxed{\frac{1}{4}}
\end{aligned}$$

We evaluated the limit above by substituting $x = 1$ into the function $\dfrac{1}{\sqrt{3x+1}+\sqrt{2x+2}}$. This is possible because the function is continuous at $x = 1$. In fact, the function is continuous at all $x \geq -\dfrac{1}{3}$.