Math 52 - Linear Algebra

David Dumas

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1. General Information

Web Page	`http://www.math.brown.edu/~ddumas/math52/`
Textbook	*Introduction to Linear Algebra*, Third Edition, by Gilbert Strang, ISBN 0961408898
Instructor	David Dumas (`ddumas@math.brown.edu`)
Office	Kassar House 114
Office Hours	Wed 12:30-1:30pm, Thurs 2:30-3:30pm, and by appointment
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2. Course Overview

In this course we will discuss vector spaces, matrices, and linear transformations.

We will start by using matrices to solve systems of linear equations. There may be no solution to a system of equations, or many solutions, and by analyzing the associated matrix we will discuss a systematic way to determine what solutions exist (if any). This will lead us to the more general idea of a matrix acting on a space of vectors, which is an example of a *linear transformation*.

We will also study geometry in vector spaces, generalizing familiar notions like distances and angles to an arbitrary number of dimensions. Here we will use matrices to represent certain geometric operations, like finding the component of a vector along a line or plane. Least-squares fitting problems provide a natural set of applications for these ideas.

Finally we will discuss how a matrix often has a set of vectors on which it acts in a particularly simple way–its *eigenvectors* (which means "own vectors", i.e. vectors owned by the matrix). Finding these vectors (and numbers associated with them, the *eigenvalues*) will make everything else about a matrix easy to determine, and will help us understand the various kinds of matrices that exist.

Eigenvectors and eigenvalues find lots of applications in other fields, and we will talk about some of them. We will use eigenvalues to solve differential equations, and to determine the long-term behavior of systems governed by probabilistic laws. As time permits we will expand on these applications and a few other concepts in linear algebra.

3. Topics

(1) Solving Systems of Linear Equations
   - Elimination and substitution
   - Elimination using a matrix
   - Matrix multiplication and its properties
   - Matrix inverses, how to find them, when they exist
   - Elimination as a factorization of a matrix (LU and LDU)
   - Permutation matrices and transposes

(2) Vector Spaces, Subspaces, and Dimension
   - Definition of a vector space and examples
   - Subspaces of $\mathbb{R}^n$
   - The Kernel or Null Space of a matrix
   - Row Reduced Form and the rank of a matrix
   - The general solution for a linear system
   - The concept of dimension and bases for vector spaces
   - Dimensions of the subspaces associated to a matrix

(3) Geometry in Vector Spaces
   - Lengths and inner products
   - Angles and orthogonality
   - Projections onto subspaces
   - The matrix of a projection
   - Application: Least squares fitting
   - Gram-Schmidt orthonormalization
   - QR factorization

(4) Determinants
   - Different definitions and formulas for the determinant
   - Fundamental properties of the determinant, relation to invertibility
   - Cofactors and the cofactor matrix
   - Determinants and geometry

(5) Eigenvalues and Eigenvectors
   - Definition of an eigenvalue, eigenvector
   - Finding eigenvectors and eigenvalues of a matrix
   - Diagonalization
   - The Jordan Normal Form

(6) Applications
   - Differential equations
   - Linear recurrences
   - Word problems, graphs
   - Markov matrices

4. Grading

Your final grade for the course will be based on your lecture attendance, homework assignments, two midterm exams, and a final exam. These components will be weighted as follows:

Attendance		4%
Homework		16%
Exam 1	Fri, Oct 7 (S1) Thurs, Oct 6 (S2)	20%
Exam 2	Wed, Nov 9 (S1) Tues, Nov 8 (S2)	20%
Final Exam	Mon, Dec 19, 2-5pm	40%

The two in-class midterms will each last one hour. The final exam will be three hours long, and its date and time are set by the registrar. You **must** be available on Monday, December 19, to take the final exam.

5. Homework Policies

Homework will be assigned weekly, with due dates announced at the time of assignment. The problems will involve material from the lectures and from the assigned reading. Do not wait until the last minute to start the homework!

Your lowest weekly homework score will be dropped, and the average of the remaining scores will account for 16% of your course grade.

Late homework will not be accepted. There will be no exceptions to this policy.

6. Attendance

Attending the lectures is mandatory; if you absolutely must miss a lecture, contact me in advance, and make arrangements to get notes and any class materials from someone in your section. You are responsible for the contents of all lectures, including any that you cannot attend.

7. Academic Honesty

All Brown University students must adhere to the standards of academic honesty set forth in the Brown Academic Code, which is available from the following URL:

`http://www.brown.edu/Administration/Dean_of_the_College/academic_code/code.html`

Particularly important are the following provisions:

- The homework you submit must be your own work. You may discuss the lectures, course material, and problems with other students, but the solutions you turn in must not be copied from anyone else.
- You must not receive assistance of any kind from other students during the midterms and the final exam.

Suspected violations of these policies will be referred to the Standing Committee. If you are found guilty of academic dishonesty, the consequences are quite severe. Loss of credit in the course and a note on your transcript is a relatively mild but not uncommon response. Temporary or permanent separation from the university may also result.