Math 535
Homework 5
Due Friday, February 27

Continue reading Chapter 4. You are encouraged to work on *all* of the exercises in the text, but you only need to turn in the following problems.

1. Ahlfors §2.2 #3, p.120, computing the integral.

2. Ahlfors §2.3 #2, p.123, showing f is a polynomial. What is the degree of f?

3. Ahlfors §2.3 #5, p.123, on the growth of the derivatives. In your "sharper" formulation, can you construct an example for which you have equality, or close to equality (same growth rate)?

4. Use the Cauchy Integral Formula to prove the Maximum Principle (see Ahlfors §3.4 Theorem 12) as follows. Let W be the domain bounded by a simple closed curve γ, and let $z_0 \in W$. Show there is a constant $C > 0$ such that

$$|f(z_0)| \leq C \sup_{z \in \gamma} |f(z)|.$$

Apply this estimate to powers $f(z)^n$, letting $n \to \infty$, to see the estimate holds for $C = 1$.

5. Recall that a function is *entire* if it is analytic on the entire complex plane. Show that if an entire function f never takes values in a line segment $[z_0, z_1]$ of positive length, then f is constant. (Use conformal maps to apply Liouville's Theorem.)