Teaching Philosophy

Heather A. Dye

Mathematics is the language used to model and solve a variety of questions. My philosophy of teaching is grounded in this viewpoint. A successful student of mathematics is able to transform a ‘real world’ question into a computational problem and solve it. To achieve this goal, I focus on the following four aspects of mathematics:

- **Definitions and Theorems**. The technical terms and rules that form the language of mathematics. We use these words constantly, both implicitly and explicitly. I emphasize ho the criteria of a definition or the hypothesis of a theorem must be satisfied in order to apply it. I support this with a variety of examples to demonstrate how the criteria of a definition or the hypothesis of a theorem are satisfied.
- **Computational Problems**. Each computational problem consists of a sequence of steps; each step consisting of the application of a theorem or rule. I discuss how to develop a large view of the problem and recognize which theorem to apply. I demonstrate how the problem satisfies certain definitions and the hypothesis of the theorem. To the students, I stress that practice will make computation easier.
- **Visualization**. This is the conceptual background of a mathematical definition or theorem. Many mathematical ideas presented in undergraduate coursework were developed to model real world problems. This aspect of mathematics should be integrated with the definitions, moving mathematics from the abstract into the concrete. It often provides inspiration for the computational aspects of mathematics, aiding students in making intuitive leaps.
- **Synthesis.** The ability to transform a ‘real world’ or word problem into a computational model and solve it. This is the most difficult aspect of mathematics for many students to master. If the students given a word problem, they must design their own computational model. Finally, they must decide what definition and theorems to apply in order to solve the resulting computations.

These aspects must be addressed when teaching mathematics. If a student can effectively apply mathematics to their life or career, I consider myself successful as a teacher. In class, I stress that students actively participate. S/he must memorize the definitions and associated concepts. They must master the computational aspect by being able to develop a strategy to solve the problem and applying it. Sufficient practice will enable them to perform a type of ‘pattern recognition’ and allow them to select the correct theorem to apply. Finally, in they must develop a sufficient conceptual basis to transform a question into a computational model.

In calculus, for example, there are many opportunities to apply these aspects of mathematics learning. The concept of a derivative is one such opportunity. This is a difficult concept for many first year college students. It involves the concepts of continuous functions and limits, which are often poorly understood. Also, a function ma not have a derivative **-** a significant difference from the process of multiplication. We consider the application of this approach to derivatives.

To introduce and define a derivative, we give the conditions necessary for a derivative to exist. I emphasize that the function is continuous and has no cusps or sharp points. The technical definition can be given, but this alone does not give the student a concept of a derivative.

Conceptually, we may discuss how to visualize a continuous function as a lines with no breaks on a graph. We may introduce a series of secant lines to this graph, and show how this progresses to a single tangent line. We ma also visually demonstrate how this is not possible when a function has a cusp or sharp point. The ideas of acceleration, velocity, and position may provide an alternative basis for a student to understand the derivative.

Computationally, there are a large number of theorems about derivatives. These must be introduced and demonstrated individually and in sequence. I try to stress that process of recognizing which of the many rules to apply is simply a matter of practice.

The last step is synthesis. If we give a student a problem about the position and acceleration of a point o tangent lines, can they recognize it's association with derivatives? We can assist the student in making this intuitive leap by examining the key associations. Discuss how to translate a word problem into the language of mathematics.

The process of learning mathematics is four tiered and requires a significant amount of participation from the student. In mathematics, the student can not passively absorb a few key phrases or a plot. In my teaching, I strive to introduce the mathematics and it's connection to the world around.