PURE AND APPLIED MODEL THEORY ABSTRACTS

A CONFERENCE AT UIC
ON THE OCCASION OF
DAVE MARKER 60^{TH} BIRTHDAY

Generic solutions of equations with iterated exponentials

Paola D'Aquino

Abstract. Abstract: Assuming Schanuel's conjecture I will show
(1) Certain exponential polynomials have a generic solution
(2) Under suitable hypothesis a variety V in $C^n x (C*)^n$ intersects the graph of exponentiation in a generic point.
(joint work with A. Fornasiero and G. Terzo)

Uniform properties of ideals in rings of p-adic power series

Mattias Aschenbrenner

Abstract. We investigate the definability of various properties of ideals (such as being radical, primary, prime, ...) generated by restricted p-adic power series depending on parameters. Some classical valuation theory makes a somewhat surprising appearance. (Joint work with Madeline Barnicle.)

Some Model Theory of Fields with Group Actions

Ozlem Beyarslan

Abstract. This is joint work with Piotr Kowalski. A G-field is a field, together with an action of a group G by field automorphisms. If an axiomatization for the class of existentially closed G-fields exists, we call the resulting theory G-TCF. If G is the group of integers, then G-TCF exists and it coincides with ACFA. More generally, when G is a finitely generated free group on n-generators, then the theory of existentially closed models is $ACFA_n$. It is also known that G-TCF exists if G is finite.

The main theorem in our paper (to appear in Proc. LMS) says that, when G is a virtually free group, then G-TCF exists. I will also talk about our recent work in progress, concerning the existence of G-TCF for a commutative and torsion group.

Date: October 19, 2018.

Erdos-Rado classes
Will Boney

Abstract. Ramsey classes are used to construct generalized indiscernibles in models of first-order theories. However, the lack of compactness means that Ramsey's theorem (and its generalizations) are not enough to build indiscernibles in nonelementary classes. Instead, order indiscernibles are built with the Erdős-Rado Theorem. We present a framework for building indiscernibles in (many) nonelementary classes: Erdős-Rado classes (and some variants). The combinatorial definition of these classes is that they satisfy a certain partition relation for building large homogeneous sets when given many colors. We connect this to building indiscernibles, present several examples (some independent of ZFC), and provide some applications.

Some applications of model theory
Elizabeth Bouscaren

Abstract. I will present some applications of pure model theory to algebraic geometry, trying to point out systematically exactly which model theoretic tools are being used. I will focus on the tools from the model theory of groups of finite rank.

Local distality and generalizations of the Elekes-Szabó theorem
Artem Chernikov

Abstract. The local distality assumption allows to recover various combinatorial properties previously established for relations definable in distal structures, including the super-Cauchy "incidence" bounds. We outline how, combined with an appropriate version of the group configuration theorem, these results can be used in certain geometric contexts (e.g. omega-stable, o-minimal) to recover a version of the Elekes-Szabó theorem. Based on joint work with David Galvin, Kobi Peterzil and Sergei Starchenko.

Notions of difference closures of difference fields.

Zoé Chatzidakis

Abstract. It is well known that a differential field K of characteristic 0 has a differential closure, which is unique to K-isomorphism. This is because the theory of differentially closed fields of characteristic 0 is totally transcendental and admits prime models over substructures.

One can ask the same question about difference fields: do they have a difference closure, and is it unique? The immediate answer to both these questions is no, for trivial reasons: in most cases, there are continuum many ways of extending an automorphism of a field to its algebraic closure. Therefore a natural requirement is to impose that the field K be algebraically closed. Similarly, if the subfield of K fixed by the automorphism is not pseudo-finite, then there are continuum many ways of extending it to a pseudo-finite field, so one needs to add the hypothesis that the fixed subfield of K is pseudo-finite.

In this talk I will show by an example that even these two conditions do not suffice.

There are two (and more) natural strengthenings of the notion of difference closure, and we show that in characteristic 0, these notions do admit unique prime models over any algebraically closed difference field K, provided the subfield of K fixed by the automorphism is large enough.

In model-theoretic terms, this corresponds to the existence and uniqueness of a-prime, or $\aleph_1$-prime, models.

In characteristic p, no such result can hold.

Tame expansions of the group of integers

Gabriel Conant

Abstract. The study of expansions of the group of integers is, in some sense, a discrete counterpart to the study of expansions of the real and complex fields. In this talk, I will start with a quick overview of work from the last few years of several authors on stable, simple, and NIP expansions of the group of integers. I will then present recent joint work with Laskowski, in which we isolate several new families of stable expansions of the group of integers by unary predicates, as well as generalizations to other weakly minimal (i.e. stable U-rank 1) abelian groups. This work also contains some new results in stability theory. For example, we show that if M is a structure with a weakly minimal theory, and A is a subset of the universe M, then the expansion (M,A) is stable if and only if the M-induced structure on A is stable (which generalizes the analogous result for strongly minimal structures, due to Pillay).

Logarithmic Hyper-Series
Lou van den Dries

ABSTRACT. In joint work with Joris van der Hoeven and Elliot Kaplan we introduced the field of logarithmic hyperseries. It has a logarithm operation that can be transfinitely iterated, and natural operations of differentiation, integration, and composition. Constructing the "correct" composition obeying the chain rule and admitting Taylor expansion is demanding. I will also discuss (partly conjectural) connections to Hardy fields and surreal numbers.

Coding of structures
Julia Knight

ABSTRACT. For some pairs of structures, one is coded in the other. There may or may not be an effective recovery procedure. We say that $\mathcal{A}$ is *Medvedev reducible* to $\mathcal{B}$ if there is a Turing operator that takes copies of $\mathcal{B}$ to copies of $\mathcal{A}$. This notion describes effective recovery.

Harrison-Trainor, Melnikov, R. Miller, and Montalbán defined a notion of *effective interpretation*. Effective interpretation gives a syntactical explanation for effective recovery. Harrison-Trainor, Melnikov, Miller and Montalbán showed that $\mathcal{A}$ is effectively interpreted in $\mathcal{B}$ iff there is one Turing operator witnessing that $\mathcal{A}$ is Medvedev reducible to $\mathcal{B}$, and another Turing operator that takes isomorphisms between copies of $\mathcal{B}$ to isomorphisms between the corresponding copies of $\mathcal{A}$. We consider several examples, including one described in Marker's basic model theory text, and one based on work in his PhD thesis.

Potential canonical Scott sentences
Michael C. Laskowski

ABSTRACT. We show how these objects can be used to prove non-Borel reducibility results among classes $Mod(\Phi)$ where Φ is either a sentence of $L_{\omega_1,\omega}$ or a countable, first order theory. We use this method to bound the Borel complexity of familiar classes, e.g., binary splitting, refining equivalence relations and jumps of abelian p-groups. We explore ancillary notions that arise, e.g., groundedness, and aim to strengthen Marker's theorem by characterizing which Φ have a Borel embedding of 'countable sets of reals' into $Mod(\Phi)$. Much of this is joint work with Douglas Ulrich and Richard Rast.

Model theory of adele rings over a number field K, as K varies
Angus Macintyre

ABSTRACT. Let A_K be the adele ring of the number field K. K is not determined by its adele ring(known for a long time), but much important information about K, such as its zeta function, is. In fact the latter is determined by the elementary theory of the adele ring, and in general elementary equivalence of adele rings implies isomorphism. I will discuss the emerging picture of the theory of the class of adele rings. (Joint work with Jamshid Derakhshan, with very recent crucial input from Udi Hrushovski)

Complexity in simple theories
Maryanthe Malliaris

Abstract. The talk will be about detecting complexity among simple theories.

Fields with Automorphisms
Alice Medvedev

Abstract. The model theory of fields with automorphisms began with ACFA, where the field has one named automorphism or, equivalently, a named action by the group $(\mathbb{Z}, +)$. Some other groups acting on fields also produce nice theories: free groups, finite groups, subgroups of $(\mathbb{Q}, +)$. Some, such as free abelian groups on two or more generators, do not. This talk will survey these results and discuss generalizations. On the algebra side, rings with endomorphisms are often of interest. On the model theory side, similar questions can be asked about other nice theories in place of fields.

Around Jouanolou-type Theorems
Rahim Moosa

Abstract. In the mid-90's, generalising a theorem of Jouanolou, Hrushovski proved that if a D-variety over the constant field C has no non-constant D-rational functions to C, then it has only finitely many D-subvarieties of codimension one. This theorem has analogues in other geometric contexts where model theory plays a role: in complex analytic geometry where it is an old theorem of Krasnov, in algebraic dynamics where it is a theorem of Bell-Rogalski-Sierra, and in meromorphic dynamics where it is a theorem of Cantat. I will report on work-in-progress with Jason Bell and Adam Topaz toward generalising and unifying these statements.

Regularity theorems in a group environment
Anand Pillay

Abstract. I will discuss recent and ongoing work with Gabe Conant and Caroline Terry (both students of Dave Marker) on regularity theorems in the context of groups equipped with a distinguished subset, sometimes under 'tameness' assumptions.

Elimination and consistency checking for difference-differential equations
Thomas Scanlon

ABSTRACT. The theory of difference fields, that is, of fields equipped with a distinguished endomorphism, has a model companion and the theory of this model companion is known to be decidable and to admit quantifier elimination in a reasonable expansion of the language of difference rings. We set out to extend the model theory of difference fields to produce algorithms to test for the solvability of difference equations in sequence rings and to solve the elimination problem for such difference equations.

Remarkably, the theory of such difference rings is undecidable, already in low quantifier complexity. However, we were able to adapt the methods behind the axiomatization of the theory of difference closed fields to produce efficient algorithms to solve both the consistency checking and the elimination problems for difference equations in sequence rings. Even more remarkably, ultraproducts of Frobenius automorphisms play a crucial role in verifying the correctness of our algorithms.

(This is a report on joint work with Alexey Ovchinnikov and Gleb Pogudin [available at arXiv:1712.01412] and an on-going project with them joined by Wei Li.)

Omega-categorical orders and homogeneous multi-orders
Pierre Simon

ABSTRACT. I will present results on the structure of some omega-categorical linear orders and applications to the classification of homogeneous structures in a language with n linear orders. (This last part is joint with Sam Braunfeld.)

O-minimal flows on compact nilmanifolds
(joint work with Y. Peterzil)
Sergei Starchenko

ABSTRACT. Let G be a real algebraic unipotent group, $\Lambda < G, M = G\Lambda$ and $\pi : G \to M$ the quotient map. In this talk we consider the topological closure $cl(\pi(X))$ for a $X \subseteq G$ definable in some o-minimal expansion of the real field, and describe it in terms of images of definable families of real algebraic subgroups of G. We also discuss C-uniformity for distributions given by definable functions $f : \Re \to G$.

Asymptotic and multidimensional asymptotic classes of finite structures
Charles Steinhorn

ABSTRACT. Abstract Asymptotic classes of finite structures and measurable structures were introduced by Macpherson and me in an effort to develop a model theory for classes of finite structures that reflects contemporary infinite model theoretic themes. Current research that generalizes these concepts to what are called multidimensional asymptotic classes and generalized measurable structures will be emphasized. This most recent work is joint with Macpherson, S. Anscombe, and D. Wolf.

Speeds of hereditary properties
Caroline Terry

Abstract. A hereditary graph property is a class of finite graphs closed under isomorphism and induced subgraphs. Given a hereditary graph property $\mathcal{H}$, the *speed* of $\mathcal{H}$ is the function which sends n to the number of distinct elements in $\mathcal{H}$ with underlying set $\{1, \dots, n\}$. Not just any function can occur as the speed of hereditary graph property. Specifically, there are discrete "jump" in the possible speeds. Study of these jumps began with work of Scheinerman and Zito in the 90's, and culminated in a series of papers from the 2000's by Balogh, Bollobás, and Weinreich, in which essentially all possible speeds of a hereditary graph property were characterized. In contrast to this, many aspects of this problem in the hypergraph setting remained unknown. In this talk we present new hypergraph analogues of many of the jumps from the graph setting, and surprising connections between these results and the model theoretic notion of mutual algebricity. This is joint work with Chris Laskowski.

Remarks on the zero sets of exponential polynomials.
Alex Wilkie

Abstract. Let K be a subfield of the complex field $\mathbb{C}$. By an exponential polynomial over K we mean a function of the form $P(z_0, \dots, z_n, e^{z_0}, \dots, e^{z_n})$ where P is a polynomial over K. In this talk I discuss complex space curves (in $\mathbb{C}^{n+1}$) given as the intersection (assumed everywhere nonsingular with respect to the variables $z_1, \dots, z_n$) of the zero sets of n such exponential polynomials. Let Ω be such a curve and let $\pi_0[\Omega]$ be its projection on to the z_0-plane. Then $\pi_0[\Omega]$ is an open subset of $\mathbb{C}$ and one expects it to be co-countable. Even this special case of Zilber's famous conjecture on the complex exponential field (which will be explained in the talk) is, as far as I know, still unknown. But I shall present a result that implies that the open set $U \cap \pi_0[\Omega]$ is dense (in U) and connected for every connected open subset U of $\mathbb{C}$. This has the model theoretic consequence that the set of reals does not lie in the σ-algebra generated by the subsets of $\mathbb{C}$ defined in the complex exponential field by existential formulas.

On model theory of universal covers
Boris Zilber

Abstract. A choice of language for universal covers of complex algebraic varieties is not a totally trivial issue, as a number of earlier examples and discussions demonstrated. Surprisingly, we found that the almost obvious natural choice of a first-order formalism suffices, while other more expressive languages are just expansions of the natural one by $L_{\omega_1,\omega}$-definable relations. Our main result is that the first-order theory of the universal cover of a complex algebraic variety $X(\mathbb{C})$ (defined over a field k) has another canonical model - the profinite universal cover (in the sense of Grothendieck) $\tilde{X}$ of X. This allows us to identify and study the étale fundamental group of X as the automorphism group of $\tilde{X}$. The categoricity problem for AEC associated with the analytic universal cover is formulated.