Math 533: Home Work 2

1. Given a set X describe the smallest the *σ-algebra* and the smallest *σ-ring* containing the individual points of X.

2. Let X be a finite or countable set and $\mathcal{E} \subset \mathcal{P}(X)$ be an arbitrary non-empty collection of subsets of X. Consider the equivalence relation $\sim$ on X where $x \sim x'$ if for every $E \in \mathcal{E}$ either $x, x' \in E$ or $x, x' \in E^c$. Prove that the σ-algebra $\mathcal{M}(\mathcal{E})$ generated by $\mathcal{E}$ is $\pi^{-1}[\mathcal{P}(Y)]$ where $Y = X/\sim$ and $\pi : X \to Y$ is the quotient.

3. Problems 1 from §1.2 (page 24).

4. Problem 3 from §1.2 (page 24).

5. Problems 4,5 from §1.2 (page 24).

6. Problems 7, 9, 10 from §1.3 (page 27).

7. Problems 8, 11 from §1.3 (page 27).