

HOMEWORK #2
DUE NOON, JANUARY 23, 2009

- (1) Give the definition of the real numbers \mathbb{R} in terms of *Dedekind cuts*, and define the inclusion map from the rational numbers to the real numbers, $f : \mathbb{Q} \rightarrow \mathbb{R}$. (That is, if a is a rational number, show how to identify a with a particular Dedekind cut.)
- (2) Define the order ' $<$ ' on Dedekind cuts. You do not need to prove anything about this order.
- (3) Define the negative of a Dedekind cut, and also addition and multiplication of Dedekind cuts. (Remember to define multiplication $a \times b$ when one or both of a and b is negative.)
- (4) Prove that if a and b are rational numbers, then addition and multiplication of a and b as Dedekind cuts agrees with the usual addition and multiplication of rational numbers. That is, if the map f is as defined in (1), prove that for all $a, b \in \mathbb{Q}$ we have:

$$f(a + b) = f(a) + f(b), \text{ and } f(a \times b) = f(a) \times f(b).$$

(In each equation above, the operation on the left hand side is the usual one in \mathbb{Q} , whereas the one on the right hand side is the one you defined above in (3).)