

HOMEWORK #4  
DUE NOON, FEBRUARY 6, 2009

- (1) Consider the sequences defined by the following formulae. In each case, (a) determine whether the sequence is convergent or divergent; and (b) if the sequence is convergent find the limit.

(i)  $a_n = \frac{n}{n+1} - \frac{n+1}{n}$ ;

(ii)  $b_n = \frac{1+(-1)^n}{n}$ ;

(iii)  $c_n = 1 + \frac{n}{n+1} \cos \frac{n\pi}{2}$ ;

(iv)  $d_n = na^n$ , for some fixed real number  $a$  with  $|a| < 1$ .

- (2) Suppose that  $(a_n)$  and  $(b_n)$  are Cauchy sequences. Prove that the following sequences are also Cauchy sequences:

(i)  $(e_n)$ , where  $e_n = |a_n - b_n|$ ;

(ii)  $(f_n)$ , where  $f_n = a_n + b_n$ ;

(iii)  $(g_n)$ , where  $g_n = a_n b_n$ .

- (3) Give an example of a pair of Cauchy sequences  $(a_n)$ ,  $(b_n)$ , where the sequence  $(h_n)$  defined by  $h_n = \frac{a_n}{b_n}$  is not Cauchy.

- (4) Show that the following series are convergent and that the sum is as given:

(i)  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$ ;

(ii)  $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{3}{4}$ ;

(iii)  $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = 1$ ;

(iv)  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n+1)}{n(n+1)} = 1$ .