Math 313 - Analysis I **Spring 2009**

HOMEWORK #5
DUE NOON, FEBRUARY 20, 2009

(1) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \lfloor x \rfloor$, so f is the function which takes a real number x to the greatest integer n so that $n \leq x$. Let $g : [1, \infty) \to \mathbb{R}$ be defined by $g(x) = \frac{1}{f(x)} = \frac{1}{\lfloor x \rfloor}$. Find $\lim_{x\to\infty} g(x)$ and prove that your answer is correct.

(2) From the definition of limit, prove that

$$\lim_{x\to 2} \frac{x-1}{x+4} = \frac{1}{6}.$$

(3) Define $h : (-1, 1) \to \mathbb{R}$ as follows: If $x = \frac{1}{n}$ for some integer $n \neq 0$, let $h(x) = 1$. For all other x, let $h(x) = 0$.

   (a) If $a = \frac{1}{n}$ for some integer $n \neq 0$, show that $\lim_{x\to a} h(x) = 0$, despite the fact that $h(a) = 1$.

   (b) Prove that if $a \neq \frac{1}{n}$ for any integer n, and $a \neq 0$, then we still have $\lim_{x\to a} h(x) = 0$.

(4) Let h be the function defined in Question 3 above. Prove that $\lim_{x\to 0} h(x)$ does not exist.