

## PRACTICE QUESTIONS FOR FINAL

These questions are intended to represent approximately how difficult and long the Final Exam will be, and also to indicate some of the types of questions that might arise. They should not be construed as a complete list of the topics that are examinable.

- (1) Show that the series

$$\sum_{n=0}^{\infty} \frac{2n+3}{(n+1)(n+2)}$$

does not converge.

- (2) Let  $f : (-1, 1) \setminus \{0\}$  be defined by:

$$f(x) = \begin{cases} \frac{e^x - 1}{x}, & \text{if } x \in (-1, 0) \\ \frac{3(x+1)}{x^2 - 4x + 3}, & \text{if } x \in (0, 1) \end{cases}$$

- (a) Prove that  $\lim_{x \rightarrow 0} f(x)$  exists.  
(b) Give a value of  $f(0)$  so that you get a function  $f : (-1, 1)$  and prove that the function thus obtained is continuous at all points of  $(-1, 1)$ .
- (3) Question 4.7, page 110 of Howie.

- (4) Assuming the Mean Value Theorem for integrals, and the Fundamental Theorem of Calculus, but not the IVT or the ordinary MVT, prove the First Mean Value Theorem:

Let  $f : [a, b] \rightarrow \mathbb{R}$  be continuous and differentiable. There is a  $c \in (a, b)$  so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- (5) Question 5.9, page 130 of Howie.