Math 330 - Abstract Algebra I **Spring 2009**

PRACTICE QUESTIONS FOR FINAL

These questions are intended to represent approximately how difficult and long the Final Exam will be, and also to indicate some of the types of questions that might arise. They should not be contrued as a complete list of the topics that are examinable.

(1) Let

$$\mathrm{GL}_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in \mathbb{R}, ab - cd \neq 0 \right\},$$

which is a group with the usual operation of multiplication.

Let H be the subset of matrices $\begin{pmatrix} a & b \\ o & d \end{pmatrix}$, with $a, b, d \in \mathbb{R}$ and $ad \neq 0$.

(a) Prove that H is a subgroup of $\mathrm{GL}_2(\mathbb{R}$;

(b) What is the identity of H?

(c) Give an explicit formula for the inverse of $\begin{pmatrix} a & b \\ o & d \end{pmatrix}$.

(2) Consider the following two elements of S_7:

x	1	2	3	4	5	6	7
α	3	4	7	1	2	5	6

and

x	1	2	3	4	5	6	7
β	2	3	1	4	5	7	6

(a) For each of the following elements of S_7, write them as a product of disjoint cycles: $\alpha, \beta, \alpha\beta, \beta^{-1}\alpha\beta, \alpha^{-1}\beta^{-1}\alpha\beta$;

(b) For each the elements in part (a), calculate their order;

(c) For each of the elements in part (a), say whether they are even or odd.

(3) Let G and H be groups, and let $\phi : G \to H$ be a homomorphism which is also onto. Let $Z(G) = \{g \in G \mid \text{ for all } g_0 \in G, g_0 g = g g_0\}$ be the center of G, and $Z(H)$ the center of H.

(a) Prove that $\phi(Z(G)) \subseteq Z(H)$.

(b) Give an example of groups G and H and a homomorphism $\phi : G \to H$ which is *not* surjective so that $\phi(Z(G)) \not\subseteq Z(H)$. [Hint: $Z(S_3) = \{1\}$]

(4) (Gallian, 14.18, p.269) Suppose that in the ring $\mathbb{Z}$ the ideal $\langle 35 \rangle$ is a proper ideal of J, and K is a proper ideal of I. What are the possibilities for J? What are the possibilities for I?

(5) (Gallian, 13.24, p.255) Let d be a positive integer. Prove that $\mathbb{Q}[\sqrt{d}] = \{a + b\sqrt{d} \mid a, b \in \mathbb{Q}\}$ is a field.