A Quick Tour of Maple

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This quick tour is intended to give you an idea of how Maple can help you in a mathematics course. For more information see *A Brief Guide to Maple*. The best way to use the material below is to try out the Maple examples at the computer, preferably with a more experienced user to help you get started.

There is no need to read all the sections in order, although it is a good idea to read 1, 2, 3, and 4 first. The section on loops can be read immediately after the section on functions. It can can also be read with profit after reading the material on series. The material on linear algebra is independent of the material on calculus. The beginning calculus student should concentrate on the first part of section 5, reading the part on series and differential equations when ready. The student who needs to use Maple for vector and matrix computations can ignore the parts which he or she is unfamiliar.

The dvi file for this document is available on the math department ftp server. To access this file go to

$$http://www.math.utah.edu/lab/ms/maple/index.html$$

and click on the **Quick Tour** link.

1. Basics

As you read through the examples below, try them out at a computer. Do each example as it is given! If you have trouble, compare what you have typed with what is written in the notes. Simple things, like forgetting a parenthesis, not matching parentheses, etc., can prevent Maple from understanding your intentions. If you still can't get Maple to do what you want it to, ask the lab assistant or someone sitting next to you for help.

Commands to Maple are written after the prompt, which on our system looks like this: `>` All Maple commands end with a semicolon, except for questions that you ask it. For example, if you want to know about factoring, you do this:

```
> ?factor
```

You can even ask for general help, which you do like this:

```
> ?
```

The most helpful part of Maple's help files are the examples. These are at the end of each help file.

Arithmetic

You can use Maple like a calculator to do arithmetic:

```
> (12.1 - 7.3)*(31 - 8.1) + 3.0^(2.1);

                                  119.9651086
```

The last number, `119.9651086`, is Maple's answer. Notice that multiplication is represented by `*` and exponentation (powers) by `^`. Sometimes you need to ask Maple to force a result into decimal form. This is what `evalf` does: it evaluates whatever you give it to "floating point" form. The quote sign " stands for the result of the last-computed expression. Note that Maple commands end with a semicolon.

Maple prefers to make exact computations when it can:

```
> 654321/123456;
                                  218107/41152
```

However, you can force it to convert the answer to decimal form ("floating point form") by using the `evalf` function:

```
> evalf( " );
                                  5.300034020
```

The quote sign " stands for the most recently computed quantity. You can do this all in one step if you like:

```
> evalf( 654321/123456 );
```

Here is one last example:

```
> sqrt(2);    # compute the square root of 2
> evalf(");   # convert to floating point form
> sqrt(2.0);  # another way of getting decimal answers
> Pi;
> evalf( " );
> evalf( Pi );
```

Try this now. Notice that if we use decimal numbers as inputs we tend to get decimal numbers as outputs.

Variables and algebra

Maple can also handle variables:

```
> F := m*a;          # the formula for force
> m := 2.1; a := 5;  # define mass and acceleration
> F;                 # compute the force
> a := 21.9;         # redefine the acceration
> F;                 # recompute the force
> a := 'a';          # clear a
> F;                 # recompute F
```

Try this now. The text following the `#` signs are *comments*; Maple ignores them.

With variables you can, of course, do algebra:

```
> factor( a^2 - b^2 );
> expand( (a + b)^5 );
> g := (a + 1)^2/(b-1)^3 + a/(b-1);
> simplify( g );
```

You can also make substitutions:

```
> subs( a = 1, g );
> simplify( " );
> subs( a = 1, b = 2, g );
> g;
```

Note that substitutions have only a temporary effect. See

```
> ?factor
> ?expand
> ?simplify
> ?subs
> ?collect
```

for more information.

Problems

1. Use Maple to define the formula for compound interest: $A = P*(1+r/100)^n$, where P is the principal, r is the annual interest rate expressed as a percentage, n is the number of years the principal is invested, and A is the amount of capital (principal plus accumulated interest) at the end of n years. Now set P to \$100, r to 5. Compute A. Then compute A with $n = 1$, $n = 5$, and $n = 10$.
2. Find the coefficient of a^2b^7 in $(2a + 3b)^9$.
3. Use the function `ifactor` to factor 123456789;

2. Functions

Many functions are predefined in Maple:

```
> sqrt(2);            # exact square root of 2
> evalf(");           # convert previous result to decimal form
> sin(Pi/2);          # pi is defined, angle is in radians
> arcsin(1/2);        # result, an angle, given in radians
> arcsin(1/4);        # !!!
> evalf( " );         # convert previous result to decimal form
> exp(2);             # the exponential of 2
> evalf(");           # convert previous result
> exp(2.0);           # get decimal result directly
> log(2);             # the natural log of 2
> evalf( log(2) );    # the natural log of 2 in decimal form
> exp(log(2));        # check!
> log(exp(2));        # another check
> ?sin                # ask about the sine function; gives more info
```

Try this now.

You can also define your own functions. Let's begin by making a function which compute the sine of an angle given in degrees. We do it this way:

```
> Sin := x -> sin( evalf(Pi*x/180) );
```

The part after the arrow tells how `Sin` will compute `x`. The definition made, let's test it:

```
> Sin(30);          # sine( 30 degrees )
```

Next let's make a function which computes the arcsine, with the result in degrees:

```
> Arcsin := x -> evalf( 180*arcsin(x)/Pi );
```

Again we test:

```
> Arcsin(1/2);      # result is in degrees
> Sin(Arcsin(1/2);  # check!
> Arcsin(Sin(90);   # another check
```

You can even define functions of several variables:

```
> distance := (u,v,x,y) -> evalf( sqrt( (u - x)^2 + (v - y)^2 ));
```

This function computes the distance from the point (u, v) to the point (x, y).

Problems.

1. The shadow cast by a flagpole is 14 feet long when the angle the sun's rays make with the vertical is 11 degrees. How tall is the flagpole?
2. Use the `distance` function defined above to see how far (11.34, 24.17) is from (−9.61, 12.49).

3. Equations

Maple can solve equations:

```
> solve( x^2 + 3*x = 2.1 );
> solve( x^3 + x = 27 );     # gives exact but complicated answer
> solve( x^3 + x = 27.0 );   # better results
```

Notice that small changes, such as using 27.0 instead of 27, force Maple to give decimal approximations. This is sometimes preferable to exact but complicated answers.

It is helpful to be able to use the results of solving one equation in another problem. Here is one example of how to do this:

```
> s := solve( x^2 + 3*x = 2.1 );  # store solutions in s
> s[1] + s[2];                    # sum of first and second solutions
> s[1]*s[2];                      # their product
> cos( s[1] );                    # apply a function to one of them
```

We can also solve systems of equations:

```
> solve( { 2*x + 3*y = 1, 5*x + 7*y = 2 }, {x, y} );
```

Of course, you can have many equations and many unknowns. If you want x and y to take the values given by solving this equation, the following will work:

```
> s := solve( { 2*x + 3*y = 1, 5*x + 7*y = 2 }, {x, y} );
> assign(s);
> x; y;        # check to see if it works
```

Even shorter is

```
> assign( solve( { 2*x + 3*y = 1, 5*x + 7*y = 2 }, {x, y} ) );
```

For more information about solve and related commands try

```
> ?solve     # for exact solutions
> ?assign    # to assign solution values to variables
> ?fsolve    # for numerical solutions
> ?factor
```

These queries will give information about useful options, e.g., the `complex` option for `fsolve`. Rememmber to look at the examples at the end of the help file if you find the general description confusing.

Problems

1. Find as many roots of $x^4 + x = 16$ as you can. (It may help to write 16 as 16.0). How many roots do you expect on theoretical grounds?
2. Find as many roots of $x^5 + x = 32$ as you can. How many roots do you expect on theoretical grounds? You may also want to see how `fsolve` works on this one. Here are some things to try:

   ```
   > solve( x^5 + x = 32 );
   > solve( x^5 + x = 32.0 );
   > fsolve( x^5 + x = 32.0, x );            # specify unknown
   > fsolve( x^5 + x = 32, x, complex );     # specify complex option
   ```

   Note that Maple may not always return the answers you expect. For example, `solve( x^2 + x = 32.0 )` *returns just one root — on my machine, with version of Maple, anwyay.*
3. Solve the system of linear equations below:

$$\begin{aligned} 7x + 3y + 8z &= 1 \\ 2x - 4y + 11z &= 2 \\ x - 7y + 9z &= 1 \end{aligned}$$

4. Use `solve` and `fsolve` to find solutions of the system

$$\begin{aligned} x^2 + 3y^2 &= 1 \\ 5x^2 + y^2 &= 1 \end{aligned}$$

   Do the results agree? Can you draw a picture to clarify the situation? Ask yourself what kind of curves $x^2 + 3y^2 = 1$ and $5x^2 + y^2 = 1$ are. For more information on how to plot two curves like this, see the *Brief Guide*.

   Graphs can tell a lot about where the solutions of one or two equations lie. For example, the solutions of $f(x) = 0$ are the places at which the graph $y = f(x)$ crosses the x axis. The solutions of the two equations $f(x, y) = 0$ and $g(x, y) = 0$ are given by the points wher two curves cross, namely, the curves defined the two equations. The next section will tell you more about graphing. Consult also the *Brief Guide*.

4. Graphs

Maple can make many kinds of graphs. Here we will look at the most basic ones. Use `?plot` or see the *Brief Guide* for more information. To plot $y = x^3 - 3x^2 + x + 1$ on the interval $-4 \le x \le 4$ use

```
> plot( x^3 - 3*x^2 + x + 1, x = -4..4 );
```

You can also set the range on the y axis:

```
> plot( x^3 - 3*x^2 + x + 1, x = -4..4, y = -4..4 );
```

With the same range on both axes you get a truer picture. Note that you can find roots by looking for the places where the graph crosses the x-axis. If you like, you can title your graph:

```
> plot( x^3 - 3*x^2 + x + 1, x = -4..4, y = -4..4,
        title = `y = x^3 - 3x^2 + x + 1` );
```

Notice that your commands to Maple don't have to be confined to a single line.

Sometimes it is useful to plot more than one curve at once. The following example illustrates this:

```
> f := sin(x);
> g := x - x^3/6;  # Taylor's approximation to sin(x)
> plot( { f, g }, x = -Pi/2..Pi/2.
         title = `y = sin(x) and its 3rd order Taylor approximation` );
```

Problems

1. Graph the the function $y = \cos(x) - (1/3)\cos(3x)$ on the interval $[-\pi, \pi]$. Then print your graph.
2. Graph $\cos(x) - (1/3)\cos(3x)$ and $\cos(x) - (1/3)\cos(3x) + (1/5)\cos(5x)$ simultaneously on the interval $[-\pi, \pi]$. Then print your graph.
3. The previous two problems establish a pattern: a sequence of functions with more cosine terms. The next one is
$$\cos(x) - (1/3)\cos(3x) + (1/5)\cos(5x) - (1/7)\cos(7x)$$
Try graphing several of these, and see if you can guess what the graph of the function with infinitely many terms is.

5. Calculus

Derivatives and Integrals

Maple can differentiate and integrate functions:

```
> n := 'n';               # make sure n is cleared
> diff( 4*cos(x^3), x );  # differentiate
> int( x^n, x );          # indefinite integral
> int( x^n, x = 0..1 );   # definite integral
```

Sometimes results are correct but not in expected form:

```
> diff( x^n, x );  # ok, but not standard
> simplify( " );   # better, but not perfect
```

You should be aware that not all integrals can be expressed in terms of elementary functions, i.e., ones made up from four arithmetic operations, extracting roots, trig functions and their inverses, the logarithm, and the exponential. Here are two examples:

```
> int( sqrt(1 + x^2), x );
> int( sqrt(1 + x^3), x );
```

Maple gives a useful answer to the first integral, but expresses the second integral in terms of yet another indefinite integral — not too helpful. Nonetheless, we Maple can use a numerical method to find definite integrals even in these "bad" cases:

```
> int( sqrt(1+x^3), x = 0..1);
> evalf( " );
```

Notice that we had to use `evalf` to force numerical evaluation. We could have done this directly, of course:

```
> evalf( int( sqrt(1+x^3), x = 0..1) );
```

You can tell whether the number just computed is "in the ballpark" by thinking about what the integral means: it is the area under a graph. Try

```
> plot( sqrt( 1 + x^3 ), x = 0..1, y = 0..1.5 );
```

to see if this is reasonable.

Series

Maple can find Taylor series of functions:

```
> series( sin(x), x = 0);      # expansion around x = 0
> series( sin(x), x = 0, 10);  # more terms
> evalf( " );                  # convert coefficients to decimals
> ?series                      # more info
```

You can also compute partial sums of any series. For example, to find

$$1 + 1/2 + 1/3 + \cdots + 1/10,$$

we do this:

```
> sum( 1/n, n = 1..10);
> evalf( " );                       # convert to decimal form
> s := n -> sum(1/i, i = 1..n);     # useful function
> s(5);                             # sum of first five terms
> s(10);                            # sum of first ten terms
> evalf( " );                       # convert to decimal
> ?sum                              # more info
```

Alternating series require a bit more thought. First we define a function which returns 1 if n is odd, -1 if n is even:

```
> sgn := n -> (-1)^n;  # easy, but inefficient
> sgn(1);
> sgn(2);
```

The rest is easy:

```
> t := n -> sum( -sgn(i)/i, i = 1..n );
> t(1);
> t(2);
> t(3);
```

Maple can compute some limits:

```
> limit(sin(x)/x, x=0);
> ?limit                    # more info
```

This occasionally works with series:

```
> w := n -> sum( 1/i^2, i=1..n);
> limit( w(n), n=infinity );
```

Differential equations

Maple can handle some differential equations. For example, to solve

$$\frac{dy}{dx} = ky,$$

the equation governing