

Lecture 1 Homework Introduction with Review

(due by Lecture 2 in Chalk FINM331 Digital Dropbox)

1. Use the completing the square technique to evaluate the expectation $E_Z[\exp(aZ)]$ with respect to the normal density $f_Z^{(n)}(z; \mu, \sigma^2)$, i.e., with mean μ and variance σ^2 , for constant a .
2. If X and Y are independent random variables with means (μ_X, μ_Y) and variances (σ_X^2, σ_Y^2) , respectively, find $\text{Cov}[X + Y, X - Y]$. Explain the significance of the difference from the case when $\sigma_Y^2 = \sigma_X^2$. (*Adapted from Rice, p. 170, problem 44.*)
3. Let $[X_i]_{n \times 1}$ be a sequence of independent RVs with $E[X_i] = \mu$ and $\text{Var}[X_i] = \sigma_i^2$. Show that if $n^{-2} \sum_{i=1}^n \sigma_i^2 \rightarrow 0$, then $\bar{X} \rightarrow \mu$ in probability. (*LLN variant, Rice, p. 188, problem 1.*)
4. Show that the standard error formula for the special acceptance-rejection technique for unit comparison function, $g(z) = 1$, is

$$\text{SE}_n^{(\text{ac})}[\hat{I}_n^{(\text{ac})}] \simeq \sqrt{\frac{1}{n} \frac{n^{(\text{ac})}}{n} \left(1 - \frac{n^{(\text{ac})}}{n}\right)}.$$

by supplying the details (*Lecture 1, p. 32, eq. (4)*).

5. Apply the Monte Carlo method to approximate the risk-neutral pricing of a European put option, i.e., with discounted payoff $\exp(-rT) \max(K - A, 0)$ where K is the strike price, T is the expiration date, A is the asset price underlying the option, and r is the risk-free interest rate; use the parameter values $\{A_0 = 110; K = 100; r = 0.055; \text{std} = 0.30; T = 1; N = 1.e5;\}$ with confidence interval percentiles $PC = [0.95, 0.975, 0.99]$ in these three cases. (See Lecture 1, p. 33).
6. Suppose that a measurement has mean μ and **std** $\sigma = 0.30$. Let \bar{X} be the average of n such measurements. How large should n be so that $\text{Prob}(|\bar{X} - \mu| < 1) \geq [0.95, 0.975, 0.99]$ in these three cases? (*Adapted from Rice, p. 189, problem 17*)
7. Select a set of X_i for $i = 1 : n$ that can be used to form a proper sample mean for the Central Limit Theorem (CLT (5)), show that the X_i are independent, and find the scaled RV that works for the CLT. Do this for
 - (a) The additive asset model of *Lecture 1, Sect. 1.4, p. 13, eq. (1)*.
 - (b) The multiplicative asset model of *Lecture 1, Sect. 1.5, p. 15, eq. (2)*.
8. For the log-jump-diffusion model (p. 42, eq. (6)), let $X_i = \log(A_{i+1}/A_i)$ and find
 - (a) mean state $\mu_X = E[X_i]$ in terms of $\{\mu_z, \sigma_z, \mu_{\log(1+\nu)}\}$ in addition to the given parameters;
 - (b) covariance in the form $\sigma_{i,j} = \text{Cov}[X_i, X_j] \simeq C \cdot \delta_{i,j}$ along with an expression for C in terms of the additional parameter $\{\sigma_{\log(1+\nu)}^2\}$ with error $O((\Delta t)^{3/2}) = o(\Delta t)$ for $\Delta t \ll 1$, where $\delta_{i,j}$ is the Kronecker delta which is 1 if $j = i$ and otherwise 0. *{Hint: It is helpful for this part to form deviations of the independent RVs, rather than the RVs themselves since the mean of the deviation from the mean is by definition zero.}*