FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2009

Lecture 2 Homework: Introduction to Exploratory Data Analysis

(Due by beginning of Lecture 3 in Chalk FINM331 Digital Dropbox.)
You must show your work, code and/or worksheet for full credit.

1. For the Poisson distribution with parameter Λ, calculate and simplify the results for

(a) $\mathrm{E}[(Z-\Lambda)^3]$;

(b) $\mathrm{E}[(Z-\Lambda)^4]$.

2. Test the MATLAB function `poissfit`, or other equivalent function in R, S, etc., to estimate the poisson parameter $\widehat{\Lambda}$ by generating `poissrnd` or equivalent 100×1 random Poisson count data with a trial parameter $\Lambda_0 = 12.37$ giving both the estimated $\widehat{\Lambda}$ and its confidence interval for $\alpha = 0.050$, 0.025 and 0.010.

3. For the discretized compound Poisson process (L2, p. 27), find the third central moment of the log-return $\mathrm{E}[(\mathrm{LR}_i - \mathrm{E}[\mathrm{LR}_i])^3]$ simplifying the exact result and then find the Δt-precision answer.

4. For the 02-January-05 to 02-January-09 get the S&P500 Index closings with symbol ^GSPC; compute the four years of log-returns for the closing; compute the mean, unbiased variance, unbiased standard deviation, coefficient of skew and coefficient of kurtosis, assuming time-independent parameters.

5. Using the data of Problem **4, produce the histogram and kernel smoothed estimation of the density on the same, professional graph of the corresponding log-returns over the four year period. On a separate graph plot the cumulative histogram for the four-year log-returns.**

6. Simulate the Δt-precision version of the (L2, p. 30), log-return discretized diffusion with compound-Poisson process model,

$$\mathrm{LR}_i \overset{\Delta t}{=} (\mu - \sigma^2/2)\Delta t + \sigma\sqrt{\Delta t} Z_i + \log(1+\nu_i)Y_i,$$

for $i = 1 : n$, where the Z_i^2 that multiplied $\sigma^2/2$ previously has been deleted since is does not appear in the stochastic calculus version of the log-return equation. For the Gaussian data values $\{n, \mu, \sigma\}$ use the values found with Problem 4, while for the compound Poisson parameters from use $\{\lambda, \mu_\nu, \sigma_\nu^2\}$ use $\{81, 0.0075, 0.018\}$ and $\Delta t = 4/n$ years. Let the component processes be independent, Z_i be a standard normal process, Y_i is a Poisson counting process and ν_i are IID. Note that $\log(1+\nu_i)Y_i$ is a simple product and not a scalar product of two vectors. Produce a histogram and kernel smoothed estimated density plotted on the same graph for the this simulated log-return process.

7. As a statistical approximation, motivate setting $Z_i^2 \simeq 1$ in Problem 6, compute the sample mean $\overline{(Z^2)}_n$ and unbiased sample variance for the Z_i^2 using the the value of n in Problem 4. Also find the theoretical mean and standard deviation of $\overline{(Z^2)}_n$ assuming the standard normal distribution for the Z_i. Explain what theorem or theorems justify the approximate result. (corrected 3/18)