FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2009

Lecture 4 Homework: NonNormal and Multivariate Exploratory Data Analysis

(Due by Lecture 5 in Chalk FINM331 Digital Dropbox)

You must show your work, code and/or worksheet for full credit.

1. Compute the quantile-quantile plot for the S&P 500 Index data log-returns of Homework 2 – Problem 4 and

(a) reference Cauchy distribution simulation with common mode and height at the mode.

(b) reference Cauchy distribution simulation with common mode and height at the mode, but use the accept-reject technique to accept only those Cauchy values within the range of the S&P 500 Index data. Do you need to renormalize the truncated Cauchy values?

Discuss the results.

2. Compute the histogram for the S&P 500 Index data log-returns of Homework 2 – Problem 4 and plot on the same graphs for each of the following:

(a) reference Cauchy PDF with common mode and height at the mode.

(b) reference Cauchy distribution simulation with common mode and height at the mode, but truncate the Cauchy PDF to be within the range of the S&P 500 Index data. Renormalize the truncated Cauchy PDF so probability is conserved. (*Hint: Can use use the Cauchy CDF to do this and do you need to readjust the height?*)

Discuss the results.

3. For the S&P 500 Index data log-returns of Homework 2 – Problem 4 pick the POT (peak over threshold value) for the left tail, then (1) extract he values less than or equal to the POT into a vector, (2) reverse the sign, and (3) sort the values in ascending order. Then,

(a) display this sorted tail vector with a histogram.

(b) use the GP distribution function `gpfit.m`, or equivalent, to fit to a GP power law; report results even if it does not work.

(c) use the fast exponential analysis function `expan.m` directly or the class modification or equivalent to fit to an exponential.

Discuss the results.

4. Let $X_0 \overset{\text{dist}}{=} \mathcal{N}(0,1)$, i.e., a standard normal, and $\mathbf{Prob}[Z_i = -1] = 0.5 = \mathbf{Prob}[Z_i = +1]$ for $i = 1\!:\!2$ be three independent RVs. Then, define $X_1 \equiv Z_1 \cdot X_0$ and $X_2 \equiv Z_2 \cdot X_0$.

(a) Prove that $X_1 \overset{\text{dist}}{=} \mathcal{N}(0,1)$, $X_2 \overset{\text{dist}}{=} \mathcal{N}(0,1)$ and that $\rho_{X_1,X_2} = 0$.

(b) Show that X_1 and X_2 are NOT independent.

Remark: this problem is from Carmona ('04) p.100, but revised for HW4, and is a standard counterexample problem that zero correlation does not imply independence, in general.