FINM331/STAT339 Financial Data Analysis – Hanson – Winter 2010

Lecture 6 Homework:

(due by Lecture 7 in Chalk FINM331 Assignments submenu)

- **You must show your work, code and/or worksheet for full credit.**
- **Justifying each non-trivial step with a reason is part of showing your work.**
- **There are 10 or more points per question if correct and best answer.**
- **Report numerical values in at least 4 significant digits (e.g., for errors use format like `%8.3e`).**

1. (20 points) **The $\mathrm{LVaR}_N(\alpha)$ to $\mathrm{LVaR}_1(\alpha)$ Conversion and k-days to Probability α Problems Revisited from HW3-Prob2:**

Let the k-day log-returns be normally distributed with mean $\mu_\ell k\Delta t$ ($\mu_\ell = \mu - \sigma^2/2$) and variance $\sigma^2 k\Delta t$, i.e.,

$$\mathrm{LR}_{i+k} \stackrel{\mathrm{dist}}{=} F_Z^{(n)}(z; \mu_\ell k\Delta t, \sigma^2 k\Delta t),$$

for $k = 1 : N$, where $\{\Delta t, \mu_\ell, \sigma^2\}$ is found from the daily log-return data.

(a) Derive the correction to the mean-less formula $\mathrm{LVaR}_N(\alpha) = \sqrt{N} \cdot \mathrm{LVaR}_1(\alpha)$. Discuss how this would affect the N-day log-Value-at-Risk in both bearish and bullish market periods with a corresponding sign of μ_ℓ.

(b) Find the quadratic correction to the mean-less answer to HW3-Prob2(b), call that answer $\widehat{k}_0^{(m)}(\alpha)$ keeping extreme tail probability

$$F_Z^{(n)}(-|\mathrm{LR}^{(m)}|; \mu_\ell k\Delta t, \sigma^2 k\Delta t) = \alpha < 0.5$$

arbitrary and not necessarily 0.25 with $|\mathrm{LR}^{(m)}|$ being the maximal or minimal log-returns from your HW3-Prob1 data. From your data, can you determine which solution of the quadratic gives the best positive solution for both maximal or minimal log-returns. Discuss how that solution compares to your HW3-Prob2(b) $\widehat{k}_0^{(m)}(\alpha)$ for both maximal or minimal log-returns $\mathrm{LR}^{(m)}$.

2. (35 points) **Maximum Likelihood Estimation:**

For this problem, assuming the market is a zero-one jump-diffusion with constant mean, volatility, jump-rate, uniform jump-amplitude distribution (a, b) parameters.

(a) Get 12/31/2005-12/31/2009 S&P500 Index daily log-return data (or any other four-year (or more) data range);

(b) Compute the log-returns and the average trading day Δt in years for 2006-2009 ;

(c) Use the MATLAB optimizer general derivative-free `fminsearch` optimizer to find the optimal estimates of 5 unknown jump-diffusion parameters by the negative log-likelihood method (see L6-pp. 45-59); for starting parameter values, use an estimate from the log-return data for Gaussian mean and variance, and set `lambda=0.5`.

(d) Report the optimal estimates of the 5 parameters to 4 significant digits and the final MLE estimate; also report any changes in the `fminsearch` options, such as `TolFun` or `Tolx` using `optimset`.

(e) Clearly state and label your results and discuss them.