FINM345/STAT390 Stochastic Calculus – Hanson – Autumn 2009

Lecture 4 Homework: Stochastic Jump and Jump-Diffusion Processes

(Due by Lecture 5 in Chalk FINM345 Assignment Submenu)

{Note dropping the Digital Dropbox}

You must show your work, code and/or worksheet for full credit.
There are 10 points per question if correct answer.

1. **Inverse Problem for Poisson Integral.**
   Find $X(t) = F(P(t))$ if

$$\int_0^t X(s)dP(s) = e^{cP(t)}\ln(aP(t)+b) - \ln(b)\ , \tag{1}$$

   where a, b and c are real constants.

2. **Solution to a Jump SDE.**
   Solve the following jump SDE for $X(t)$, $\mathrm{E}[X(t)]$, and $\mathrm{Var}[X(t)]$,

$$dX(t) = aX^2(t)dt - \frac{bX^2(t)}{1+bX(t)}dP(t)\ , \tag{2}$$

   where $a < 0$, $b > 0$ and $c > 0$ are constants such that $\mathrm{E}[P(t)] = ct$, while $X(0) = x_0 > 0$, with probability one.
   {*Hint: seek a transformation $Y(t) = F(X(t))$ for some F such that $Y(t)$ satisfies a constant coefficient SDE. The answer may be left as a Poisson distribution sum.*}

3. **Solution to another Jump SDE..** Solve the following Poisson jump SDE for $X(t)$ and $\mathrm{E}[X(t)]$:

$$dX(t) = a\sqrt{X(t)}dt + b\left(b + 2\sqrt{X(t)}\right)dP(t),$$

   where $\mathrm{E}[P(t)] = \lambda_0 t$ and $X(0) = x_0 > 0$, with probability one, while λ_0, a and b are real constants.
   (*Hint: Find a power transformation to convert the SDE to a constant coefficient SDE.*)

4. **For Jump-Diffusion SDE, Find Coefficients Transformable to Constant Coefficient SDE.** Show that the (Itô) jump-diffusion SDE for $X(t)$,

$$dX(t) = f(X(t))dt + bX^a(t)dW(t) + h(X(t))dP(t)\ , \tag{3}$$

   can be transformed by $Y(t) = F(X(t))$ to a **constant coefficient SDE**, where b and $a \neq 1$ are real constants, and $X(0) = x_0 > 0$, with probability one. In a proper answer, derive the power forms of $f(X(t))$ and $h(X(t))$ from the constant coefficient SDE conditions. Also, what is the answer when $a = 1$?