FINM345/STAT390 Stochastic Calculus – Hanson – Autumn 2009

Lecture 6 Homework (HW6): Marked Jump-Diffusions Stochastic Calculus Continued

(Due by Lecture 7 in Chalk FINM345 Assignment Submenu)

{Note: Dropped the Digital Dropbox}

You must show your work, code and/or worksheet for full credit. There are 10 points per question if correct answer and negative points for missing homework sets.

Corrections are in ***Red*** as are comments, November 4, 2009

1. Simulate $X(t)$ for the log-normally distributed jump amplitude case with mean $\mu_j = \mathrm{E}[Q] = 2.74$ and variance $\sigma_j^2 = \mathrm{Var}[Q] = 1.38$ for the linear jump-diffusion SDE model for a population in a seasonal environment $\mu(t) = 0.1085\sin(2\pi t - 0.75\pi)$, $\sigma(t) = 0.0485 - 0.0233\sin(2\pi t - 0.75\pi)$, $\lambda(t) = 3.98 - 0.0115\sin(2\pi t - 0.75\pi)$ per year, $\nu(t, Q) = \exp(Q) - 1$ with Q normally distributed, $X(0) = 0.5$, $t_0 = 0$, final time $T = 2.0$ years, $N = 2,000$ time-steps per state for M = 4 states.

{*Hint: Modify the linear mark-jump-diffusion SDE simulator* ***linmarkjumpdiff09fig1.m*** *linmarkjumpdiff09fig1.m on pp. L6-p(29-37) and found on Chalk/CourseDocuments. For consistency, you may want to change the* `randn` *to* `normrnd` *with a similar sequential call pattern as used with* `binornd`*.*}

2. Show that the **Itô mean square limit** for correlated bond $B(t)$ and stock $S(t)$ price portfolio with diffusions $W_B(t)$ and $W_S(t)$, respectively, at time t, so that

$$dW_B(t)dW_S(t) \overset{\mathrm{dt}}{=} \rho(t)dt\ , \tag{1}$$

is valid, where

$$\mathrm{Cov}[\Delta W_B(t_i), \Delta W_S(t_i)[] \simeq \rho(t_i)\Delta t_i$$

for sufficiently small Δt_i and with absolute integrability of the correlation coefficient $\rho(t)$, i.e.,

$$\int_0^t |\rho|(s)ds < \infty.$$

Are there any special treatments required if $\rho = 0$ or $\rho = \pm 1$? You may use the bivariate normal density in (B1.44) or Table B1 of selected moments of preliminaries Online Appendix B.

3. Provide background justification for the values of the mark deviation sums $\mathrm{E}[(\sum_{i=1}^{k}(Q_i - \mu_j)^m]$ for $m = 1:4$ moments in Theorem 5.9 by proving
Lemma - Expectation of Mark Deviation Sums:
If the set $\{Z_i : \ i = 1\!:\!\infty\}$ is a set of zero-mean IID random variables, i.e., $\mathrm{E}[Z_i] = 0$ and $\mathrm{Cov}[Z_i, Z_j] = \mathrm{E}[Z_i^2]\delta_{i,j} \equiv \overline{Z^2}\delta_{i,j}$, show that by induction (where necessary) that

(a) $\mathrm{E}[\sum_{i=1}^{k} Z_i] = 0$;

(b) $\mathrm{E}[(\sum_{i=1}^{k} Z_i)^2] = k\overline{Z^2}$, where $\overline{Z^2} \equiv \mathrm{E}[Z_j^2]$, any j;

(c) $\mathrm{E}[(\sum_{i=1}^{k} Z_i)^3] = k\overline{Z^3}$, where $\overline{Z^3} \equiv \mathrm{E}[Z_j^3]$, any j;

(d) $\mathrm{E}[(\sum_{i=1}^{k} Z_i)^4] = k\overline{Z^4} + 3k(k-1)(\overline{Z^2})^2$, where $\overline{Z^4} \equiv \mathrm{E}[Z_j^4]$, any j.

4. Prove the corollary for the variance of $X(t)$ from the solution of the linear mark-jump-diffusion SDE (6.13) on L6-p15 (5.42, p. 142, textbook):

Corollary 6.2var. Variance of X(t) for Linear SDE.

Let $X(t)$ be the solution of (6.13) (5.42, textbook) with $\overline{\nu^2}(t) \equiv \mathrm{E}[\nu^2(t, Q)]$ of (6.16:L6-p15) (5.45, textbook) Then

$$\mathrm{Var}[dX(t)/X(t)] \stackrel{\mathrm{dt}}{=} \left(\sigma_d^2(t) + \lambda(t)\overline{\nu^2}(t)\right) dt$$

and

$$\mathrm{Var}[X(t)] = \mathrm{E}^2[X(t)]\left(\exp\left(\int_{t_0}^{t} \mathrm{Var}[dX(s)/X(s)]ds\right) - 1\right). \tag{2}$$

Be sure to state what extra conditions on processes and precision are needed that were not needed for proving Corollary 6.2 on L6-p45 (Corollary 5.13, p. 146, textbook).