FINM345/STAT390 Stochastic Calculus – Hanson – Autumn 2009

Lecture 7 Homework (HW7): Compound-Jump-Diffusion Distributions, Black-Sholes & Merton Option Pricing

(Due by Lecture 8 in Chalk FINM345 Assignment Submenu)

{Note: Dropped the Digital Dropbox}

You must show your work, code and/or worksheet for full credit. There are 10 points per question if correct answer and negative points for missing homework sets.

November 9, 2009

1. Construct the MATLAB (or other reasonable code) for simulating the Normal-Uniform Hybrid Mark random variables discussed on L6-p2 & L6-p3.

(a) Present the simulation results in a histogram with a reasonable bin size. Let the simulation sample size be $N = 5\text{e+}3$. Use the sample parameters, $a = -0.0947$, $b = +0.1096$, $\mu_n = 2.448\text{e-}4$, $\sigma_n = 1.121\text{e-}2$ and $p_u = 0.60$.

(b) Also, compute and report the simulated mean, standard deviation, coefficient of skewness and coefficient of kurtosis.

{*Hint: For the truncated normal part of the hybrid distribution, you will need to simulate a sufficient number of normal variates in* (a, b), *rejecting those outside the interval, so that the accepted total is at least* N.}

2. Let $dW_b(t)$ and $dW_s(t)$ be correlated diffusion differentials with correlation coefficient $\rho(t)$ to precision dt. Let $dW_p(t)$ be uncorrelated with $dW_s(t)$, i.e., $dW_p(t)dW_s(t) \stackrel{\mathrm{dt}}{=} 0$.

(a) Show that if $dW_b = \alpha(t)dW_s + \beta(t)dW_{p)}$, then find expressions for the deterministic constants $\alpha(t)$ and $\beta(t)$ in terms of $\rho(t)$.

(b) If the corresponding increment versions of the differentials are $\Delta W_b(t)$, $\Delta W_s(t)$ and $\Delta W_p(t)$, then evaluate

$$\mathrm{E}\left[(\Delta W_b)^2(t) \cdot (\Delta W_s)^2(t)\right] \tag{1}$$

in terms of $\rho(t)$ and Δt.

{*Please note that this is Guoquan's problem, given with a hint for solving.*}

3. Formally show by differentiation and limits that Black-Scholes formulas (7.16-7.17) on L7-p36 satisfy the Black-Sholes PDE problem (7.15) on L7-p34 including the showing the limiting final conditions on L7-p35 (note that the formulas are singular in the limit). Do this both for the European call and put prices.

4. The Greeks (Sensitivity Coefficients): From the Black-Scholes formula,

(a) The deltas of both calls and puts, i.e.,

$$\Delta_C = \frac{\partial C}{\partial s}(s,t) \;\; \& \;\; \Delta_P = \frac{\partial P}{\partial s}(s,t).$$

(b) The vegas of both calls and puts, i.e.,

$$\mathcal{V}_C = \frac{\partial C}{\partial \sigma_0}(s,t;K,T,r_0,\sigma_0) \;\; \& \;\; \mathcal{V}_P = \frac{\partial P}{\partial \sigma_0}(s,t;K,T,r_0,\sigma_0).$$

5. **Black-Scholes European Option Pricing:** Let $S_0 = \$100$, $r_0 = 2.25\%$ per year (p.a.) and $\sigma_0 = 21\%$ without dividends.

   (a) Compute the Black-Scholes call prices for strike prices $K = 80:5:120$ in US dollars for each exercise time $T = 0.25\!:\!0.25\!:\!1.00$ years.

   (b) Similarly, compute the European put price directly from Black-Scholes pricing for puts.

   (c) Plot the call prices versus the strike prices K with T-values as the parameter for each respective curve using different symbols or other distinct markings.

   (d) Separately plot the put prices similarly.

   (e) Verify the Put-Call Parity using the Black-Scholes put and call prices, plotting the percentage errors relative to the Black-Scholes put prices versus K and parameterized by T on one plot.

   Comments:

   - You may code your own programs, or use modifications of the Global Derivatives Black-Scholes MATLAB code, as long as you verify it,
     `http://www.global-derivatives.com/code/matlab/BlackScholesEuro.m`
     with instructions in the m-code preface and further explanations in
     `http://www.global-derivatives.com/index.php?option=com_content&task=view&id=52&Itemid=31`
   - It is suggested that when your main MATLAB m-code calls other functions, you can avoid path and structure problems by naming your main program a function and copy-pasting all available called–functions at the end of main so that they are proper subfunctions (name is still function).