FINM345/STAT390 Stochastic Calculus – Hanson – Autumn 2009

Lecture 8 Homework (HW8): More Merton Option Pricing and JD Financial Applications

(Due by Lecture 8 in Chalk FINM345 Assignment Submenu)

{Note: Dropped the Digital Dropbox}

You must show your work, code and/or worksheet for full credit. There are 10 points per question if correct answer and negative points for missing homework sets.

Corrections are in ***Red*** as are comments, November 15, 2009

1. **Computation Using Merton's (1976) Jump-Diffusion Model for European Options:** Compute the European call and put option prices for the following model:

$$dS(t) = S(t) \cdot ((\mu - \overline{\nu}\lambda) \cdot dt + \sigma \cdot dW(t)) + \sum_{i=1}^{dP(t)} S(T_i^-)\nu_i,$$

where here ν is an IID random variable (i.e., the jump-amplitude is its own mark, which in Merton's 1976 model is log-normally distributed), T_i is the ith jump-time with jump-amplitude ν_i, $\boldsymbol{\mu = \mu_0 + \overline{\nu}\lambda}$ here is the total jump-diffusion mean rate coefficient, σ is just the diffusion volatility coefficient, and λ is the number of jumps per year. All coefficients are constant. You can use Kevin Cheng's Global Derivatives Merton Jump-Diffusion MATLAB function code (Caution: watch for errors):

`http://www.global-derivatives.com/code/matlab/MertonJumpEuro.m` ,

which calls his Basic Black-Scholes European option MATLAB function multiple times:

`http://www.global-derivatives.com/code/matlab/BlackScholesEuro.m` .

It is recommended, to avoid path problems, that you merge these functions into one MATLAB m-file, with a beginning driver function for handling input and output.

   (a) Compute results for both call and put;

   (b) Do this with the two values of Vol given below, meaning a total of 4 cases (2 Vol and 2 call/put) ;

   (c) Compare results with respect to Vol;

   (d) Use the following input: S_0 = \$95, K = \$100, $r_0 = 0.025$ (2.5%), $T = 0.25$ years is the time to maturity, `Vol` $= \{0.20 \;\; \& \;\; 0.30\}$, or $\{(20\%) \;\; \& \;\; (30\%)\}$, respectively, are two possible total jump-diffusion volatilities of the asset ($\text{Vol}^2 \equiv \sigma_d^2 + \lambda_j(\mu_j^2 + \sigma_j^2)$), where j subscripts denote jump and d denotes diffusion), `Gamma` $= 0.60$ (60%) is percent of jump-diffusion volatility explained by jumps (Gamma$\equiv \sqrt{1 - \sigma_d^2/\mathtt{Vol}^2}$), `Jumps` $= 25$ is the number of jumps per year (λ_j), MaxIter $= 100$ is the number of total jumps included.

2. Jump-Diffusion Monte Carlo Option Pricing: Compute the European call and put prices for log-uniformally distributed jumps for the stock model,

$$dS(t) = S(t)((\mu_0 dt + \sigma_0 dW(t)) + \sum_{i=1}^{dP(t)} S(T_i^-)\nu(Q_i,)$$

where $\mathrm{E}[P(t)] = \lambda_0 t$, $\nu(Q) = \exp(Q) - 1$. You may use put-call parity to compute the puts from the call results. Use $S_0 = \$95$, $K = \$96 : 105$, $r_0 = 0.025$/year, $T = 0.25$years, $\sigma_0 = 0.20$ & 0.30, $a = -0.09$, $b = 0.11$, and $\lambda_0 = 25$/year.

(a) Compute the call and put prices for the range of values of K and the two values of σ_0. Recall put prices can be calculated by parity.

(b) Graph the call prices C versus moneyness S_0/K parameterized by both values of σ_0 on the same plot. Do the same for the put prices.

(c) You may use the Zhu-Hanson 2005 Monte Carlo code, ***CMC05ATOCV.m*** on Chalk/CourseDocuments, but it was originally set up for online input and tabulated data, so needs to be modified for graphs and batch data loops and graphs. This Monte Carlo simutlation code is conditioned for antithetic-thetic (AT) and optimal control variate (OCV) variance reduction.

3. Let log-return compound process at exercise be $\widehat{\mathcal{S}}(t) = \sum_{j=1}^{P(t)} Q_j$ corresonding to the return process $\mathrm{CP}(t, Q) = \sum_{j=1}^{P(t)} \nu(Q_j)$ with $\nu(Q) = \exp(Q) - 1$ and $\mathrm{E}[P(t)] = \Lambda(t)$.

(a) Show that $\mathrm{E}\left[\widehat{\mathcal{S}}(t)\right] = \Lambda(t)\mathrm{E}_Q[Q]$.

(b) Show that $\mathrm{E}[\mathrm{CP}(t, Q)] = \Lambda(t)\overline{\nu}$, where $\overline{nu} = \mathrm{E}_Q[\nu(Q)]$.

(c) Show that $\widetilde{\mathrm{CP}}(t, Q) \equiv \sum_{j=1}^{P(t)} \nu(Q_j) - \lambda_0\overline{\nu}t$ is a martingale.

(d) Let $\widetilde{\mathcal{S}}(t) = \widehat{\mathcal{S}}(t) - \Lambda(t)\overline{\nu}$ and show that $\exp\left(\widetilde{\mathcal{S}}(t)\right)$ is an exponential martingale, i.e., $\mathrm{E}\left[\exp(\widetilde{\mathcal{S}}(t)) \middle| \widetilde{\mathcal{S}}(0)\right] = \exp\left(\widetilde{\mathcal{S}}(0)\right)$.

4. Jump-Diffusion European Option Prices are Bigger Than Black-Scholes? Show that

$$\mathcal{C}^{(\mathrm{jd})} \geq \mathcal{C}^{(\mathrm{bs})} \quad \& \quad \mathcal{P}^{(\mathrm{jd})} \geq \mathcal{P}^{(\mathrm{bs})},$$

independent of the Q-mark distribution and for the same arguments $(S_0, T; K, r_0, \sigma_0)$, i.e., $\mathcal{C}^{(\mathrm{jd})} = \mathcal{C}(S_0, T; K, r_0, \sigma_0)$, etc. Also, discuss the monetary reasons in words why this is so. {*See the theorem statement on L8-p49.*}