**FINM345/STAT390 Stochastic Calculus – Hanson – Autumn 2009**

**Lecture 9 Homework (HW9): Stochastic Volatility**

**(Due by Lecture 10 in Chalk FINM345 Assignment Submenu)**

**{Note: Dropped the Digital Dropbox}**

**You must show your work, code and/or worksheet for full credit. There are 10 points per question if best correct answer and negative points for missing homework sets.**

Corrections are in ***Red*** as are comments, November 22, 2009

**1.** Show that the nonsingular, explicit, exact solution [L9-p16:(9.13)],

$$V(t)=e^{-\overline{\kappa}_v(t)}\left(\sqrt{V_0}+0.5\int_0^t e^{\overline{\kappa}_v(s)/2}(\sigma_v dW_v)(s)\right)^2, \quad (1)$$

when $\sigma_v^2(t) = 4\kappa_v(t)\theta_v(t)\ \forall t$, is a solution satisfying the mean-reverting, square-root diffusion [L9-p16:(9.2)],

$$dV(t)=\kappa_v(t)(\theta_v(t)-V(t))dt+\sigma_v(t)\sqrt{V(t)}dW_v(t), \quad V(0)=V_0>0, \quad (2)$$

by the Itô calculus or by increment expansion of $\boldsymbol{V(t)}$ solution (9.13) in the limit of $dt$–precision.

**2.** Simulate a solution to the SV-SDE in Eq. (1) using the the more robust finite difference form that is less sensitive to numerical problems for small values of the variance, i.e., use

$$V^{(\varepsilon)}(t+\Delta t)=\max\left(V^{(\varepsilon)}(t)+\kappa_v\cdot(\theta_v-V^{(\varepsilon)}(t))\cdot\Delta t+\sigma_v\cdot\sqrt{V^{(\varepsilon)}(t)}\cdot\Delta W_v(t),\ \varepsilon_v\right), \quad (3)$$

$V^{(\varepsilon)}(0)=V_0$, where $\varepsilon_v$ is some small positive cutoff to avoid very small variance values and $\Delta t$ is the simulation time step.

In addition, consider the deterministic solution for constant coefficients,

$$V^{(\mathrm{det})}(t)=V_0\cdot\exp(-\kappa_v\cdot t)+\theta_v\cdot(1-\exp(-\kappa_v\cdot t)), \quad V^{(\mathrm{det})}(0)=V_0>0, \quad (4)$$

Let $V_0=0.25$ per year, $\kappa_v=2.0$ per year, $\theta_v=0.15$ per year, $\sigma_v=0.20$ per year, $T=2$ years and $N=8000\cdot T$ giving $\Delta T$ and $\varepsilon_v=\sqrt{\Delta t}$ to ensure $\Delta t/\varepsilon_v \ll 1$ for $\Delta t \ll 1$. Then

(a) Simulate the cutoff modified solution $V^{(\varepsilon)}(t)$ in (3) with the given parameters.

(b) Simulate the deterministic solution $V^{(\mathrm{det})}(t)$ in (4), i.e., with $\sigma_v = 0$, on [0,T] using the same time steps.

(c) Plot the simulations of $V^{(\varepsilon)}(t)$ and $(\Delta SV)$, and the difference $\delta V(t) = V^{(\varepsilon)}(t) - V^{(\mathrm{det})}(t)$ on a single plot with appropriate legend. Also, print out the maximun of absolute value of $\delta V(t)$.

(d) Discuss the results. Also, justify why $\{V_0, \kappa_v, \theta_v, \sigma_v\}$ all are in units *per year* when $t$ by convention has those units; and tell what are the units of $\sigma_s$, the Black-Scholes stock price volatility are.