Abstract

Dirac operators on foliations with invariant transverse measures

We extend the groundbreaking results of Gromov and Lawson on positive scalar curvature and the Dirac operator on complete Riemannian manifolds to Dirac operators defined along the leaves of foliations of non-compact complete Riemannian manifolds which admit invariant transverse measures. We prove a relative measured index theorem for pairs of such manifolds, foliations and operators, which are identified off compact subsets of the manifolds. We assume that the spectral projections of the leafwise operators for some interval $[0, \epsilon]$, $\epsilon > 0$, have finite dimensional images when paired with the invariant transverse measures. As a prime example, we show that if the zeroth order operators in the associated Bochner Identities are uniformly positive off compact subsets of the manifolds, then they satisfies the hypotheses of our relative measured index theorem. Using these results, we show that for a large collection of spin foliations, the space of positive scalar curvature metrics on each foliation has infinitely many path connected components.