A Symbol Calculus for Foliations

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The classical Getzler rescaling theorem is extended to the transverse geometry of foliations. More precisely, a Getzler rescaling calculus, as well as a Block-Fox calculus of asymptotic operators, is constructed for all transversely spin foliations. This calculus is an appropriate tool for a better understanding of the index theory of transversely elliptic operators on foliations. The main result is that the composition of asymptotic pseudo-differential operators is again an asymptotic psuedo-differential operator, and includes a formula for the leading symbol. Our formula is more complicated due to its wide generality but is quite similar to Getzler's formula, and it simplifies notably for Riemannian foliations. In short, we construct an asymptotic pseudo-differential calculus for the "leaf space" of any foliation. In subsequent papers we will give a Getzler-like proof of a local topological formula for the Connes-Chern character of the Connes-Moscovici spectral triple of a foliation, as well a (semi-finite) spectral triple associated to a covering foliation, yielding an extension of the Atiyah-Singer covering index theorem to covering foliations.