

Math 215 HW for Wednesday March 18

Question 1. Determine all possible groups with 4 elements. *I.e.*, take a set with 4 elements: $\{e, \alpha, \beta, \gamma\}$ in which e is the identity. Write down the possible multiplication tables using these four elements, and the fact that e is the identity and that each row and column contains each element exactly once. You should find that there are really only two possibilities (up to permuting α, β, γ)

Question 2. Write down the multiplication table for $\mathbb{Z}/8$ and show that the subset consisting of the congruence classes of $\{1, 3, 5, 7\}$ is a group. Compare with your answer to question 1.

Write down the addition table for $\mathbb{Z}/4$ and compare to question 1.

Question 3. Look at the axioms for a group. (See the Wikipedia article or your notes). Suppose we replace the existence of inverses by the axiom:

For every element a of G there exist elements b and c such that $ab = e$ and $ca = e$.

Prove (being clear about which axioms you use at each step!) that then $b = c$ (This is known as “left inverses = right inverses”).