

Math 215 MT 2 sample questions

Q1. Suppose that T is a subset of \mathbb{N} , such that $1 \in T$ and $\forall n \in \mathbb{N}, \{1, \dots, n\} \subset T \Rightarrow \{1, \dots, n+1\} \subset T$; prove that $T = \mathbb{N}$. Hint: induction.

Deduce that if $S \subset \mathbb{N}$ is a non-empty subset, it has a smallest element.

Q2. Prove that if S is a subset of \mathbb{Z} , $S \neq \emptyset$ which is closed under addition and subtraction, then $\exists d \geq 0$ such that $S = d\mathbb{Z}$.

Q3. Prove that if $a \in \mathbb{Z}$ and $n \in \mathbb{N}$, then a is invertible mod n if and only if a and n are coprime.

Q4. Let p be a prime number. Show that $x^2 - py^2 = 0$ has no non-trivial solutions.

Q5. Prove that if G is a group, then $\forall g \in G$, the function $\lambda_g : G \rightarrow G$, $\lambda_g : h \mapsto gh$ is *bijective*.

Q6. Write down the multiplication table of $\mathbb{Z}/12\mathbb{Z}$. Which elements are invertible? write down the multiplication table of just the invertible elements. Do the same for the invertible elements of $\mathbb{Z}/8\mathbb{Z}$. What do you notice?