1. Axioms about Addition and Multiplication – *Field* axioms

(1) (Commutativity) For all a and b, $a + b = b + a$ and $ab = ba$;
(2) (Associativity) For all a, b and c,$(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$;
(3) (Distributivity) For all a, b and c, $a(b + c) = ab + ac$;
(4) (Zero) There is an element 0 such that for all a, $a + 0 = a = 0 + a$;
(5) (Unity) There is an element 1 such that for all a, $a1 = a$; furthermore, we assume that $1 \neq 0$
(6) (Subtraction) For all a, the equation $a+x = 0$ has a unique solution $x = -a$. Similarly, the equation $a + x = b$ has a unique solution $b - a$.
(7) (Division) If $a \neq 0$, The equation $ax = 1$ has a unique solution $x = a^{-1}$. Similarly the equation $ax = b$ has a unique solution $x = b/a = ba^{-1}$.

2. Order Axioms

(1) (Trichotomy) Either $a = b$, $a < b$ or $b < a$, and only one these holds.
(2) (Multiplication Law) If $c > 0$, then $ac < bc$ if and only if $a < b$,
if $c < 0$, then $ac < bc$ if and only if $b < a$;
(3) (Addition Law) $a < b$ if and only if $a + c < b + c$;
(4) (Transitivity) If $a < b$ and $b < c$, then $a < c$.

Axioms i)xi) are true in the real numbers $\mathbb{R}$ and the rational numbers $\mathbb{Q}$. Axioms i)vi) and viii)x) are true in the integers $\mathbb{Z}$.