

Sample Math 215 Final Questions

You should also make sure you know how to do the questions on the mid-terms and homework.

Q1. Define the following concepts:

- (1) \sim is an equivalence relation on X .
- (2) X and Y are sets with the same cardinality (“are equipotent”).
- (3) X is a denumerable set.

Q2. State the Pigeonhole Principle.

Q3. Sketch the proof that \mathbb{R} , the set of real numbers, is uncountable.

Q4. Decide if each of the following statement is TRUE or FALSE. If FALSE, give an example showing it is FALSE.

- (1) If A is countable, then A is infinite.
- (2) If A is infinite, then $|\mathbb{N}| \leq |A|$.
- (3) If $|X| = |Y|$ and $f : X \rightarrow Y$ is injective, then f is a bijection.
- (4) The relation on \mathbb{Z} defined by $x \sim y$ if and only if $x + y$ is odd for integers x, y is an equivalence relation.
- (5) If $f : X \rightarrow Y$, then $\{f^{-1}\{y\} \mid y \in Y\}$ is a partition of X .

Q5. The dyadic rationals is the set $D = \{\frac{a}{2^n} \mid a \in \mathbb{Z}, n \in \mathbb{N}\}$. Prove that D is countable.

Q6. Show that if n is odd, 9 divides $8^n + 1$.

Q7. Consider the statement: If $a < b$ then $f(a) < f(b)$ (Here f is a function $\mathbb{R} \rightarrow \mathbb{R}$).

- (1) What is the contrapositive of the statement?.
- (2) What is the converse of the statement?.
- (3) What is the negation of the statement?

Q8. Prove by induction that the sum of the squares of the odd integers satisfies:

$$1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 = \frac{n(2n - 1)(2n + 1)}{3}$$

for all $n \geq 1$.

Q9.

- (1) State the division theorem.
- (2) State the well ordering principle.
- (3) Let $A \subseteq \mathbb{Z}$ be a subset which contains both zero and at least one positive integer, and which is closed under addition and subtraction. Using the first two items, show that there is a positive integer d such that A consists of all the multiples of d .