

HOMEWORK

Week 1, due Wednesday January 21.

- Q1:** (1) Show that given a topological space X , that the operation of assigning to a continuous map $\pi : T \rightarrow X$ (“a space over X ”) its sheaf of sections Γ_π is a covariant functor from spaces over X to sheaves of sets over X .
- (2) Then show that this has a left adjoint. *I.e.*, there is a functor L from sheaves of sets on X to spaces over X , such that for every sheaf \mathcal{F} on X and space over X , $\pi : T \rightarrow X$ there is a natural 1-1 correspondence between maps of sheaves of sets $\mathcal{F} \rightarrow \mathcal{G}$ and continuous maps $L(\mathcal{F}) \rightarrow L(\mathcal{G})$ of spaces over X . (Hint: L is the étale space)
- (3) Show that if \mathcal{F} is a presheaf on X , and $|\mathcal{F}| \rightarrow X$ is the associated étale space, then the sheaf of sections of $|\mathcal{F}|$ over X is the sheafification of \mathcal{F} .
- Q2:** Let X be the complex plane minus the origin. Consider the presheaf $\mathcal{O}_X^{\text{an}}$ such that $\mathcal{O}_X^{\text{an}}(U)$ is the ring of holomorphic complex valued functions on U , and let $\mathcal{O}_X^{\text{an},*}$ be the subpresheaf of nowhere vanishing holomorphic functions. Show that these are both sheaves. and that sending $f \mapsto \exp(f)$ is a map of sheaves. Show that it is surjective (or epi) as a map of sheaves, but *not* as presheaves – *i.e.* there is a section $f \in \mathcal{O}_X^{\text{an},*}(X)$ *not* in the image of $\mathcal{O}_X^{\text{an}}(X)$
- Q3:** Hartshorne CH 2 1.16
- Q4:** Hartshorne CH 2 1.20

Week 2, due Wednesday January 28.

- Q1:** Hartshorne CH 2 2.8
- Q2:** Hartshorne CH 2 2.10
- Q3:** Hartshorne CH 2 2.11

Week 3, due Wednesday February 4.

- Q1:** Let k be a field, and $\mathbb{A}_k^2 = \text{Spec}(k[x, y])$. The origin 0 in \mathbb{A}_k^2 is the closed point corresponding to the maximal ideal (x, y) . Let $X \subset \mathbb{A}_k^2$ be the open subset $\mathbb{A}_k^2 \setminus \{0\}$. X , together with the restriction of the structure sheaf of \mathbb{A}_k^2 is a scheme. What is the smallest affine open cover of X that you can find?
- Q2:** We know that for any ring R , the R -valued points of \mathbb{A}^2 are simply pairs of points in R - *i.e.*, R^2 . Show that for any k -algebra R , the R -valued points of X are a subset of R^2 and *characterize this subset*.
- Q3:** Let G be a group. For a fixed positive integer n , we can consider the set of homomorphisms $G \rightarrow GL_n(R)$. This is a covariant functor from rings to sets. Show that it is represented by a scheme.