Solving a quadratic equation
a case study

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A pesky problem

Your paycheck has been held up, and they keep asking,

"Are you really a mathematician?"

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If that doesn't convince the admin type, what will?

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2. $x^2 - 2x + 1 = 0$ (more of the same)
3. $x^2 - 3x - 1 = 0$ (sort of fancy... just right!)

Grab your calculators:

A picture may be worth a thousand words, but is it worth a thousand bucks?
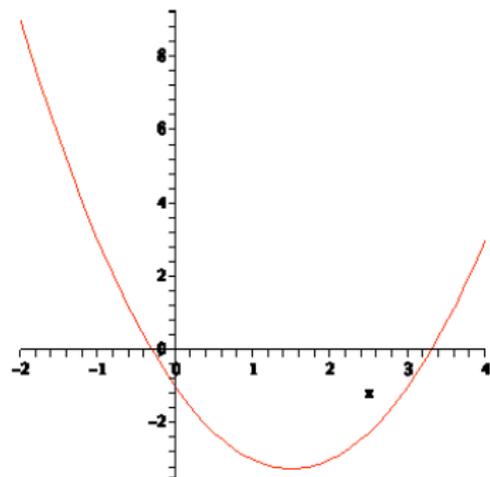
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Let's try! If they buy this, we are done. So plot $y = x^2 - 3x - 1$

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"Give the solution to 10 decimals, and we'll show you the money!"

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"Give the solution to 10 decimals, and we'll show you the money!"

"Oh, for @#%& sake!"

factor, factor, complete...

$$0 \ = \ x^2 - 3x - 1$$

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\begin{aligned}
0 &= x^2 - 3x - 1 \\
0 &= x^2 - 3x + (-3/2)^2 - (3/2)^2 - 1
\end{aligned}
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0 &= (x - 3/2)^2 - 9/4 - 4/4
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0 &= (x - 3/2)^2 - 13/4
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Progress

Now let's solve it:

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Think this is enough to get the money?

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$x = 3/2 + \sqrt{13/4}$, or

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and $x = 3/2 - \sqrt{13/4}$, or

$x = -0.302775637731994646559610633735247973125648286922623106$

Mathematical Proof

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Give them the Magic Formula,

$$ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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and tell them to try this first next time...