COLLEGE MATHEMATICS: SUGGESTIONS ON HOW TO TEACH IT

Prepared by the

Committee on the Teaching of
Undergraduate Mathematics

Mathematical Association of America

March, 1979

The printing and distribution of this pamphlet have been made possible by a bequest from the late Professor Carl B. Allendoerfer and a gift from his wife.

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FOREWORD*

Nobody should deny that it is important that the teacher teach effectively; however, some may well ask why it is necessary to choose this moment to emphasize the importance of the effective teaching of mathematics. Are we not extraordinarily effective in producing young mathematicians? Indeed, are we not embarrassingly successful? Do we not have a surfeit of young Ph.D.'s in mathematics today, who are unable to find jobs suiting their talent, training and appetite? Of course the answer to these rhetorical questions is that the community of academic mathematicians performs satisfactorily in reproducing its own kind, but this is by no means the only criterion of success in teaching, certainly not the only task we have as teachers of mathematics. One should add, parenthetically, that it is even doubtful whether we should always credit ourselves with actually producing young mathematicians. Nevertheless, we may fairly claim that we do encourage intelligent young people to become mathematicians—once they have succeeded in overcoming the many and varied obstacles that lie between elementary school and graduate mathematics.

However, the main point to be made is that we are far less successful in teaching effectively those who are not destined to become professional mathematicians; and these, of course, constitute the vast majority of our clientele as teachers of undergraduate mathematics. We must certainly take into consideration the potential users of mathematics, since the main argument for the importance of mathematics today is precisely the ubiquity of its many applications. Thus future biologists, economists, architects, town planners, ecologists, . . . (as well as physicists, chemists, statisticians) need to understand mathematics and to be able to apply mathematics to their own disciplines. This means they must learn to think mathematically—a very different matter from solving mathematical problems of a familiar form by the use of a specified technique—and must know how to set up mathematical models of non-mathematical situations, and how to judge the appropriateness of those models.

We also have a responsibility, as teachers of mathematics, to cater to the future citizen, the future adult. Mathematics, we

*This eloquent personal statement was written in 1971 by Professor Peter Hilton, Battelle Memorial Institute, for the CUPM report, "Suggestions on the Teaching of College Mathematics." It is no less appropriate today.

should not be ashamed to proclaim, is part of our great cultural heritage. A person is deprived by being blind to it—as most people are today. We cannot begin to justify our own privileged position as scholars—which means that we are concerned with creating mathematics, learning mathematics, and teaching mathematics—unless we strive to awaken in as many people as possible, irrespective of their chosen vocation, an awareness of the nature of our science, and its significance for our civilization, material and spiritual.

Thus, we see that it is a matter of great social importance that we should teach as well as we possibly can. Moreover, when considering the problem of teaching those who are not professionally oriented towards pure mathematics, our task becomes more complicated because we must prepare ourselves to answer broader questions about the significance of the particular topics under discussion. This clearly calls for deeper understanding, on the part of the teacher, of the subject matter itself than that characterized by the ability to get the right answer to questions posed by others and to teach students a similar skill. It also calls, of course, for an understanding of the nature of the students' difficulties, actual and potential, and the ability to encourage the student to express his difficulties without shyness or fear of ridicule. Further, it is no easy task, even if one understands why the student fails to understand, to find a way to improve his understanding. Simple repetition, for example, is very unlikely to be effective! Thus, to sum up, it would seem that all the most difficult problems of teaching mathematics are really to be found in the task of teaching undergraduates who, while they may very well plan to use mathematics in their special field of interest, are not going to be professional mathematicians.

So far we have been giving what might be called "unselfish" reasons for teaching mathematics effectively. However, it is very easy to give "selfish" reasons too! It is a common experience of those who have spent much of their professional life in teaching that the necessity to give an exposition of a particular topic leads to a substantial clarification of their own ideas. This can happen even when the topic is relatively elementary. In particular, we often see the significance of a mathematical idea with which we are currently concerned in our research far better when we fully comprehend the origins from which it springs. Those origins may, of course, be within mathematics itself or outside mathematics. We may also argue that a selfish reason for teaching mathematics well is that, in our own enlightened self-interest, we should do so in

order to keep our jobs! Moreover, and less cynically, there is enormous enjoyment in good teaching, and ineffective teaching is a notorious source of severe frustrations.

Granted that the teaching of mathematics is important, it follows that it must be seen by the future mathematicians to be important. This implies that the faculty must set a good example to the graduate student by the quality of their own teaching and the attention which they pay to improving it. For the graduate student will usually choose some outstanding faculty member as a model, so that the attitude towards teaching of the most influential members of the faculty will largely determine the attitude of the student. If, for example, a senior professor of conspicuous research talent shows an impatience, or even disdain, for undergraduate teaching, or, perhaps, the teaching of future biologists, then this attitude is likely to be transmitted to graduate students. On the other hand, enthusiastic and effective teaching not only generate in the student joy in mathematics, they also are likely to help the student to become a fine teacher in turn. Thus both graduate and undergraduate students will derive healthy, positive attitudes towards teaching from the example of an outstanding scholar who clearly responds with imagination and enthusiasm to the challenge of undergraduate teaching.

By a similar argument it follows that the faculty must give attention to the nature and quality of the teaching done by the graduate students in their capacity as TA's. It must often seem to a TA that the teaching done is simply part of a formal device for acquiring a bare subsistence wage, and that the only really important activity is research. This view should not be inculcated in the graduate student. It must be realized that this period as a graduate student is a period of apprenticeship in the craft, and that the craft comprises, in essential fashion, both research and teaching. The student's teaching, and the interest of the faculty in it, must neither of them be perfunctory. The faculty must help the TA in very many ways to lay the groundwork of a good teaching technique and to recognize that teaching constitutes an integral part of professional competence. It is thus of importance to try to set down procedures and criteria of effectiveness which are likely to lead to good teaching. It is to such a task that the authors of this pamphlet have addressed themselves.

INTRODUCTION

In the Spring of 1977 Henry Alder, President of the Mathematical Association of America, appointed the Committee on the Teaching of Undergraduate Mathematics (CTUM) and listed as one of its charges the preparation of a pamphlet offering guidelines for successful instruction. It is our hope that this pamphlet will serve as a useful resource for all college mathematics teachers.

The ideas offered here are suggestions only. We would not presume to propose any specific prescription for good teaching. Also, most of the advice given in this booklet is based on the traditional class-lecture format for teaching mathematics, for this is the format within which most prospective readers will be expected to operate. This does not mean that we believe the traditional way to be necessarily the best way. Many colleges and universities are now experimenting with new styles and patterns of teaching undergraduate mathematics, and although there has not been time for a full evaluation of the results, these developments are well worth watching. Other new ideas are sure to appear. In general, teachers of mathematics who are serious about their craft and alive to its possibilities will always be searching for ways of improving their methods.

A substantial amount of the material in this booklet has been drawn from the 1972 report of the Committee on the Undergraduate Program in Mathematics (CUPM), "Suggestions on the Teaching of Undergraduate Mathematics," which in turn was based on material produced by departments of mathematics at several universities.

The fact that so large a part of this booklet is concerned with everyday teaching practices should not be taken to mean that they are more important than everything else. For one thing, we should note that competence is a *sine qua non* for instruction; the person who does not know mathematics, and well, will not teach it well. Mastery of teaching techniques without mastery of subject matter will not result in good instruction. However, the person with subject matter knowledge but without an effective means of communicating it will fail as a teacher. This pamphlet does not address the question of subject matter mastery; we assume it. These pages are written for those who know their material and are looking for suggestions as to effective ways to present it.

CTUM has produced a companion pamphlet, "Training Programs for Teaching Assistants in Mathematics," suggesting means for improving the preparation and supervision of graduate students in their role as teaching assistants. Copies of either pamphlet may be obtained from the Mathematical Association of America, 1529 Eighteenth Street, N.W., Washington, D.C. 20036.

I. CONDUCTING THE COURSE

Planning

"And gladly wolde he lerne, and gladly teche."

Chaucer, *The Canterbury Tales*

To be effective in the classroom, a teacher must ordinarily spend some time thinking about every course being taught long before the onset of classes. The goals of the course should be clear, and some key decisions affecting the likelihood of attaining those goals should be made.

It is important to consider how the course fits into the curriculum. For instance, if the course is prerequisite to other courses, there will be a certain minimal body of material that the instructor must cover thoroughly lest the students experience unnecessary hardship when they move on. A teacher should decide in advance on the approach to the major topics in the course and on the level at which they will be presented. This requires that the teacher know something about the interests and preparation of the students who will make up the class. A class consisting entirely of mathematics majors should be treated quite differently from a class consisting, say, of business students.

It is good to have an overall conception of the course, and the text should be examined with this in mind. Before classes begin, one should note sections of the book deserving emphasis as well as sections that can be omitted without serious loss if time pressures develop. Particularly for courses with several sections taught by different people, it is desirable to prepare a day by day outline prior to the start of the course together with a suggested set of daily homework assignments. For many-sectioned courses, this outline should clearly indicate the minimum material to be covered in all sections.

The preparation of any supplementary material and location of source books are also best done in advance. As the teacher, you may wish to enrich your own background for the course: not only the mathematical content but also historical background, interesting applications, and recent developments. Students like to hear about these matters, but few instructors are ready to present supplementary material of this kind without some advance preparation.

If there are useful reference books available to the students, an annotated bibliography should be prepared and distributed to them. It may be desirable to put some books on reserve in the library or to notify local bookstores that certain books are being recommended.

Time spent in advance planning is amply rewarded by a more smoothly running, more consistently paced course and by heightened student respect for the seriousness of the enterprise. On the other hand, planning should not be too rigid or detailed. Classes, like individuals, differ from one another and should be treated accordingly.

Good Beginnings

> *"No bubble is so iridescent or floats longer than that blown by the successful teacher."*
>
> Sir William Osler, quoted in Harvey Cushing, *Life of Sir William Osler*, Volume II

There are several housekeeping details that should be attended to at the first class meeting.

1. Announce the title and number of the course.

This will ensure that the students in the room are in the right place.

2. Announce the prerequisites of the course.

Make it very clear that only students having satisfied the prerequisites with passing grades are admitted. If there are no specific prerequisites for your course, but students will need to have certain computational skills, you might give a short diagnostic test which they can grade themselves. You can warn them that scores below a certain cut-off point indicate that they will have difficulty; that scores below another indicate that they should seek remediation before entering the class.

3. Write on the blackboard your name, office number, office hours, and possibly your telephone number.

Office hours are important. Some of your most effective moments as a teacher may occur when you are helping students in your office. Leaving these encounters on a hit or miss basis can cause frustration for the students when they do not find you in your office and irritation for you when they do. Scheduling office hours at staggered times may make the opportunity available to more

students; it should also be made clear that you can be seen by appointment if the office hours are inadequate, and you should remind students frequently of your availability. If nothing else, these reminders reassure the students of your interest in helping them learn.

4. Explain your policies on homework, examinations, and grading.

To avoid misunderstandings you might give the students a handout which explains these policies. Students have a right to know how their work will be evaluated, and they are more likely to meet your expectations if they know what they are.

5. Write on the blackboard the title(s) and author(s) of the text(s).

In case a text has several editions, specify the edition you want the students to have. Also announce any basic reference books you believe the students may want to buy or consult in the library.

6. State the objectives of the course.

Students will have a better perspective of the role and objectives of the course if some time is spent outlining the topics to be covered. Briefly discuss how the course relates to others in mathematics or to other disciplines. A good way to whet the students' appetite is to state in the first lecture some problems they can't do, but will be able to do by the end of the course. A statement of three or four such problems—which might be picked from the text—has the effect of creating an immediate interest in the course.

7. Announce your preferred policy for taking notes in your course.

Students are often in doubt about whether to try to take verbatim notes at the risk of not understanding each point as it is presented, to take no notes at all, or to use a strategy somewhere in between. Advice appropriate to your style of teaching will be welcome.

8. Allow ample time to whet the students' appetite for the course.

Do not spend more than about 10 minutes on the bookkeeping details (items 1,2,3,4,5, and 7 above). Thus 4 can, for example, be taken care of by a promise to discuss testing and grading at a later meeting of the course. It is important to use the first-day flush of enthusiasm as effectively as possible to get the students excited about the course. Always use the full time of the first class meet-

ing; dismissing a class early at its first meeting can set an undesirable tone.

One direct way of exhibiting your interest in the students is to get to know them quickly. Here are a few suggestions that may help you identify names with faces.

(i) Take roll at the beginning of class for the first few meetings. In doing so you may wish to note where each student is seated. Most of the students will be in approximately the same locations all term.

(ii) Return assignments to individual students at the beginning of each class period.

(iii) During an examination the names of those not yet known to you may be obtained directly from the papers. Requiring that all completed examination papers be handed directly to you offers an excellent opportunity to associate names with faces.

Students tend to show an initial burst of enthusiasm. The first two weeks may be best used to develop a topic that is new or useful to most of them. What is learned first is often learned well, so a topic that will reappear frequently through the course should probably be taught first.

Some teachers think that beginnings must be slow. However, the first few weeks should set the pace for the entire course. One good reason for this is that some students must decide on the basis of this period whether to stay in class or move up, down, or out, and should not be misled by an atypical pace.

The Class Session

"The ideal condition
Would be, I admit, that men should be right by instinct;
But since we are likely to go astray,
The reasonable thing is to learn from those who can teach."

Sophocles, *Antigone*, Ode II

Preparation

Prepare before every class meeting! This does not mean that you should write out a complete program or even a detailed outline beforehand. It does mean that before each session with your students you should have a clear idea of what you wish to cover and

how you wish to cover it. Don't try to lecture off the top of your head. The results can be disastrous. However, do not be an inflexible slave to your plan. If an interesting point arises unexpectedly, you should be able to grant it a short digression.

While preparing for class, look several sections ahead in the text for the sake of perspective. Prepare more material than you expect to need in an hour, but do not feel obliged to cover everything you have prepared.

Try to begin each class with a short discussion of material with which the students already feel relatively comfortable, rather than plunging directly into new territory. A brief résumé of the previous topic may help to provide continuity. When starting new material, you should state the objective of the discussion first, that is, give a survey of what the class is trying to accomplish before going into details. By inviting questions on recent material at the start of the period, you may find yourself provided with a natural springboard for the topic of the day. Associating the new material with the old helps students to enter the new on a firmer footing. To guard against the danger of answering questions on homework in class that are of interest to only a few students, ask the class how many students want the problem to be done; if only a very few (or no) others request it to be done, don't do it, and advise the student asking the question to see you after class.

Try to plan out the rough structure of the class in advance. If it will take 35 minutes to discuss new material for example, make certain to leave a 35 minute block for the discussion.

When presenting a new idea, explain how it fits into the general scheme of the course. A common complaint among students is that although they may understand this or that piece of information, they are not just sure what it "means". Sometimes they express this by saying that the idea is unmotivated: they don't know why they are studying it, how it impinges on other ideas, or how it will be used. Sometimes we will have to expect students to learn on faith, but this should not be allowed to become standard policy.

It is often useful to allocate blocks of time for discussion of the fundamental concepts of the course, how they relate to one another, and their applications in other disciplines. This is particularly true in classes made up primarily of nonmath majors.

Plan to ask frequent questions of individual students. One of the many advantages of this technique is that students are given a chance to participate in the development of new concepts. When the questions bear on material already covered, they can remind students of what they should know. Questioning also gives the

teacher an opportunity to rework concepts when students show that they are unsure about them. Problems exposed in this way are probably shared by several students, if not by a majority of the class, so time spent on them is not wasted. Another advantage is that this tactic helps to keep students alert. You may recall from your own experience how hard it was to daydream in a foreign language class when the professor might call on you at any moment. Call on all the students. If you call on just the bright ones, the poorer students will be discouraged; if you call on just the slow ones, the better students will be bored and may develop a false sense of security. Yet this kind of questioning must be done delicately, and must not seem like an inquisition to the students. They must not come to fear the classroom encounter.

Avoid lecturing for more than a short period if you possibly can. Students get many long lectures, but they seldom have the appetite or powers of concentration to follow them properly. In particular, it defeats the purpose of having a small number of students in a class to lecture them as though they were a vast multitude.

It makes for a much livelier class to introduce new material on the basis of answers and conjectures drawn from the students. The best balance between this "discovery" method of teaching and more conventional approaches will depend on the circumstances. You should in any case avoid giving the class the impression they are being talked down to.

Instructional Techniques

Here are some specific suggestions for conducting the class.

1. Arrive promptly in your classroom, preferably a minute or two before the scheduled time of the lecture.

This allows you to take care of preliminaries such as returning homework.

2. Look the students in the eye.

Every good teacher wants rapport with his or her class, but it is amazing how many instructors give their lucid explanations to the blackboard, the walls, a window, or a point about one foot over the students' heads.

3. Speak clearly, distinctly, and in a voice loud enough to be heard by everyone.

It helps to pick out one or two students and address yourself directly to them, watching their faces for signs of understanding. Of course the same students should not always be chosen for this role.

4. Modulate your voice.

Speaking for long periods of time at the same pitch, amplitude, and speed has a tendency to put the listener to sleep. To avoid this, speak sometimes in a high voice and sometimes low, speak sometimes loud and sometimes soft, speak sometimes fast and sometimes slow.

5. It is not necessary that someone be talking every second of the class.

A pause after a sentence can give the class an opportunity to think of what you have said and avoids the tensions a rapid-fire presentation frequently engenders. In particular make sure that you give students in the class a fair amount of time to consider any questions that you raise. Otherwise they come to treat the questions as rhetorical.

6. Encourage the students to ask questions.

At the beginning of class, it is frequently desirable to ask the class whether they have any questions on the homework. After asking for questions, wait, while looking at the class, to give the students time to formulate their ideas.

7. When a question is asked, make sure everyone has heard the question before proceeding.

Repeat the question if there is any doubt. This is particularly important if someone in the front of the room asks the question. Do not carry on a discussion with someone in the first or second row which cannot be heard, much less understood, by those in the rear of the room. Rephrase the question if you feel the original was not clear or precise.

Even well-prepared students will often not ask questions for fear of appearing foolish before their classmates. It is not *you* they are worried about, but their peers. The problem is a serious one. There are several things that can help.

8. Encourage students to speak up by asking many questions and having as many students as possible recite.

In this way timid students will be made to feel that their own questions will attract little notice.

9. Encourage conjectures and do not ridicule inept questions or wrong answers.

Give the students the feeling that they are all on an equal footing in your esteem. You can learn from their mistakes. Above all,

avoid sarcasm in any form. Nothing can damage your relationship with a class faster than sarcasm, however warranted it may seem.

Sometimes an embarrassing situation occurs when some students laugh because they consider a question which has been asked by a student to be ridiculous. In that case, tell the class something like this: "I am sure that many of you had the same question in your minds but did not dare to ask it. Mr. Smith deserves a lot of credit for having had the courage to ask this question and thereby giving all of us the opportunity to discuss it. It is always better to ask a question than to be afraid to ask it." Then proceed to answer the question.

10. Listen to your students!

When someone volunteers an answer to one of your questions, you may realize that the approach is incorrect. Resist the urge to interrupt. Instead, try to understand what is being said, acknowledge any merit in it, determine the misconceptions, and tactfully point them out. Then let the student try again, or give someone else a chance. Many instructors misinterpret a question before it is completely formulated. After you have tried to answer a question, ask whether the student is satisfied with your response.

11. Whenever appropriate let the class guess the answer to a problem.

By guessing the student becomes committed to the problem being solved, and thereby a much more interested participant in the solution of the problem.

12. Strive for as much informality in the classroom as your own personality and the circumstances will allow.

Studies have shown that students usually perceive teachers who conduct an informal class as actually smarter and more knowledgeable in the subject than those who are stiffer and more formal. This impression is certainly often erroneous, but it has to be reckoned with. At any rate, informal give-and-take can be more pleasing for both you and your students. An overly casual, super breezy type of informality in demeanor, speech, or dress, however, should be avoided, as it does not generate either the student attitudes nor the professional image best suited to a learning situation.

13. Show enthusiasm for the subject matter.

Never display boredom. Enthusiasm and boredom are extremely communicable.

14. Try to inject humor into your presentation.

Humorous anecdotes related to a specific topic almost invariably will cause the student to remember that topic much more easily.

15. Try to smile occasionally.

A smile immediately creates a positive attitude among your students both towards you and towards the subject material being covered. A pleasant mien and a frequent smile help in establishing a congenial teacher-class relationship.

16. Never give way to anger in a classroom.

If you have a misguided, disgruntled, or possibly dyspeptic student in your class, ask this student to see you in your office. Always deal with such cases one-to-one, never in a classroom.

17. Be courteous to your students and take other steps to earn their respect.

Unfailing courtesy to students in tone and manner will elicit respect and courtesy from them that creates the classroom atmosphere most conducive to learning. Take a few other simple steps to earn the respect of your students, such as not offending students by your language or dress.

18. Include interesting pieces of historical information in your lecture.

Exciting bits of information on the history and evolution of mathematical concepts of topics in your course can greatly add to the students' interest. When covering arithmetic progressions or covering some of Gauss' many discoveries, you might tell the story how, as a student in the first grade, he was able to answer the teacher's assignment to find the sum $1 + 2 + 3 + \ldots + 100$, as soon as the teacher had finished asking the question.

19. Don't be defensive when you make a mistake.

No one is perfect, and an impression of integrity is more important than an impression of omniscience. Request help from the students and correct the error together. If someone asks a question you cannot answer, do not try to cover your ignorance. Try to work out the answer with the help of the class, or promise to bring it to the next meeting.

20. End your class as close to the scheduled time as possible.

Avoid keeping students late. Many of them may have another class immediately following yours, and they don't want to be late, especially if an examination is scheduled for the following class. In

any event, it is almost impossible to hold students' attention in overtime.

21. Do whatever you do well, carefully, and completely.

It is far better to omit something than to rush your presentation, especially at the end of a lecture when the attention span of your students may already have been exhausted.

An obvious but important consideration is that your classroom technique must be adapted to the class you are teaching. A calculus class of 300 engineers, a small honors section in calculus, a remedial class in sub-freshman mathematics, and an advanced undergraduate course for majors must all be handled differently. Many of the suggestions mentioned above obviously apply mainly, but not entirely, to smaller classes and to less mature and less well-motivated students.

Teaching Large Classes

Large lecture sections present special problems. In order to have more students taught by experienced teachers, many colleges schedule large lecture sections of mathematics classes. In this case some of the instructional techniques discussed in this booklet will need to be modified, and with these modifications the teaching of a large class can be a quite satisfactory experience. Although there are advantages to a small class, there are going to continue to be circumstances where large classes are necessary.

In teaching a large class, you may not learn the names of all your students, and your teaching assistants will take over many of the duties of helping students outside of class, so it is especially important that you be aware of the students' problems and project this concern in the classroom. The teaching assistants can help you to be aware of areas of particular difficulty for the students, so be certain to communicate with them regularly.

In this class you should make an even greater point of reviewing the material covered in the previous lecture, of presenting examples in detail, and of putting the material in perspective.

For a class of about seventy-five taught in a large classroom you can still use the blackboard, but you will need to write a little larger, not use the bottom part of the board, and speak a little louder. In a lecture hall an overhead projector is frequently preferable. In this case you should spend some time practicing with it prior to the first class meeting. Although it is unlikely that students can be called upon to recite in any systematic way, students

in a large section should be actively involved and the instructor should try to make everyone a participant in the proceedings.

Some excellent teachers are very uncomfortable teaching a large class, and it may be a good idea for you to present a guest lecture in a large section before volunteering to teach one.

Essential Blackboard Techniques

1. Write clearly, carefully, and slowly.

Press the chalk firmly against the blackboard. Never write with the chalk perpendicular to the blackboard. By all means, avoid squeaking with the chalk. Breaking it in half usually stops the noise. Writing slowly not only helps to ensure clarity, but also gives a better psychological tone to the presentation.

2. Repeat in a (particularly) loud, clear voice what you are writing as you write on the blackboard.

It may help to imagine that you have a blind student in your class.

3. Begin writing at the top of the blackboard panel on the left, move down that panel, and then proceed to the next panel.

Avoid erasing the first panel until all available panels have been used. You should not skip around the blackboard, placing equations haphazardly here and there. If your room is equipped with a triple-layer blackboard, use the board sandwiched between the front and back boards first. When finished, push it up and pull the front board down, so that what was just written is still in sight. After using the front board, push it up and use the back board, leaving the front board visible above. Then move to the next panel if there is one.

If you have more than two rows of students in the class and the room has a level floor, do not write all the way to the bottom of the board, because those farther back will not be able to see. If there is a desk in front of the class, keep it clear of such large objects as lecterns and briefcases.

4. As much as possible, avoid standing in front of what you are writing while you are writing.

Step back from the blackboard after you have completed writing so that everyone can read what you have written.

5. Plan ahead, so that your writing will remain approximately the same size (except when emphasis is desired).

Some teachers work with eraser in hand, using it to simplify and correct as they go along. This is guaranteed to irritate students who are taking notes. Instead of revising by erasure, draw a distinct line through the offending terms and write the improved forms above them.

6. Write in detail complete and precise statements of what you propose to prove or examine.

Define in writing any special symbols used in the solution; summarize at the conclusion. Your students' notes will consist almost entirely of what you have written on the blackboard; if you have omitted some crucial "ifs" and "thens", then that is the way the student will probably remember the statement. Students' notes should fill, not reinforce, gaps in their recollections of a lecture.

7. Write solutions to problems on the board exactly as you would expect students to write them.

Solutions on the blackboard should be considered models of solutions for students to follow. This means that you should write everything in proper form.

8. When solving a problem at the blackboard, write a complete statement of the problem or give a precise reference.

If the problem comes from the text, write the page and problem number on the board before starting the solution. Define in writing any variables used in the solution. Unless the routine computations are prohibitively long, work the problem to completion in order to give your students practice in recognizing a solution and to give yourself an opportunity to illustrate checking and interpreting results.

9. Important items may be emphasized by outlining them in chalk.

This is particularly true of facts you want your students to memorize, such as the formulas for the derivatives of the trigonometric functions. (Although some things may need to be learned cold—this is particularly true of definitions—students should always be encouraged to think problems through from first principles or basic results instead of immediately appealing to some memorized rule.)

10. Don't ever *write* an incorrect statement on the board and *tell* the students that this is not correct.

The students very likely will copy the incorrect statement from the blackboard, but fail to note your statement that it is not correct.

11. Don't rush out after class.

Linger for a few minutes so that students may have an opportunity to ask questions. When all the students have finished taking notes, erase the blackboard as a courtesy to the instructor of the following class.

Textbooks

Although textbooks are a nearly indispensable part of teaching, they can also be sources of difficulty. The temptation to condemn the chosen text and hand out an alternate set of notes should be strongly resisted. The text can be changed for later classes, but once it has been designated for a term it is best to live with it for that term and compensate for its deficiencies by lecturing especially well. Criticism of the text before a class undermines its authority, persuades the students that they have been victimized in being asked to buy the book, and generally erodes morale. Of course, specific errors in the text should be corrected, but general carping should be avoided.

There is also the possibility that a text is so well written that it seems to leave little for the teacher to do. A teacher can always clarify and augment the text with additional examples, alternative derivations and proofs, and applications. The teacher should take advantage of the text's comprehensiveness to give more attention to the individual problems of the students. The classroom time you gain from this should be considered an excellent opportunity. You can break the class into small groups of students and have them work on new problems, for example.

Students, especially in the more elementary courses, usually have difficulty reading mathematics, even well-written mathematics; actually reading a page or two aloud in class, with comments on important ideas, details that need to be filled in, the structure of the argument, etc., may be of great educational value. This is teaching students to read mathematics—one of the most valuable services a mathematics teacher can perform.

In any case, the text and syllabus provide cohesion, and extensive deviations are not wise. This is particularly true in multisection classes. Uniformity among sections is essential if students are not to have extra trouble in the next course.

Visual Aids

Even the teacher who relies mainly on blackboards may occasionally be able to make good use of an overhead projector. Com-

plicated drawings can be made in advance, and for some elementary courses professionally made transparencies are available.

Modern devices for the inexpensive reproduction of printed and written material offer many opportunities. Material complementary to the text, model answers to homework or test questions, lists of references, and special tables can now be put into students' hands quickly, neatly, and at very low cost. There is less justification than ever for wasting class time on the laborious copying down of such things.

There now exists a wide range of movies on mathematical topics, and these might be used to enrich your lectures or, from time to time, to substitute for them. For example, the film "Mathematical Induction" by Leon Henkin could be shown in an off-period for students who need a review of that principle. Films of this kind are regularly reviewed in the *American Mathematical Monthly*. A large number of films produced by the Mathematical Association of America may be rented or purchased from several film distribution agencies. For details write to the Mathematical Association of America, 1529 Eighteenth Street, N.W., Washington, D.C., 20036. If there is an office of audio-visual services on your campus, assistance in arranging to show films may be obtained there.

Assignments

"There is no royal road to geometry."

Euclid, quoted by Proclus,
Commentary on Euclid, prologue

The Rationale

Some instructors give no assignments at all on the assumption that students will work on their own and ask questions about points which they do not understand. Instructors like this may have forgotten their own ways as undergraduates. Courses compete with one another for student time, and courses in which assignments are not given regularly are very likely to suffer. After a while the students reach a point where they are so far behind that they cannot ask even the most basic questions, and recovery becomes almost impossible.

1. Give frequent assignments, with a reasonable selection of routine exercises and more difficult problems.

Many practices are possible. You can require that the assignments be handed in, and grade one or two problems without telling the

student in advance which ones those will be. Another possibility is not to ask that all assignments be handed in, but to collect two or three serious problems a week, which should then be graded and returned. Some teachers ask their students to accumulate homework in a notebook which is called in for checking from time to time.

2. Choose homework problems carefully.

For many students these will be the only problems they consult. When making out assignments, be sure to cover a particular topic on at least two assignments, but preferably on three; avoid having students work on each topic only once.

3. Collect homework at the *beginning* of the class in which it is due.

Do not accept homework at the end of a class. Do not accept late homework except in case of illness of the student or death or other emergency in the family.

4. Count the homework for part of the course grade.

Counting homework as part of the course grade provides an additional incentive for the students to take homework seriously.

5. Evidence indicates that daily assignments give the best results.

These should be returned by you at the following meeting. This method has a number of advantages:

(i) You have continual opportunities to see how your students are doing.
(ii) Short assignments can be corrected quickly and are likely to encourage students to study the course.
(iii) Students' correct ideas are reinforced while they are still fresh, and their wrong ones are corrected before they have had time to become habitual.
(iv) The student becomes accustomed to regular and systematic study and will not wait until the day before a test to begin studying.
(v) Returning assignments frequently helps you to learn students' names and to maintain person-to-person relationships with them.
(vi) Students acquire a clear idea of what sorts of problems they should be able to do and build a set of solved problems for review.

6. Occasionally weekly assignments are preferable.

You may wish to assign problems occasionally that either require more time than the usual ones or are such that you would like to have your students give quite a bit of thought to them. In these cases, you might announce that such problems are due a week after they have been assigned. If you prepare and distribute a dittoed assignment sheet, the student can keep track of which problems have been finished and which are presenting special difficulties. Some students prefer to study in large units of time instead of a short period each day. Assignment sheets allow students to work ahead and to budget study time in their own ways.

7. Suggest discussion among students.

Part of an instructor's task is to encourage students to talk about mathematics as well as to listen and to read about it. One way to do this is to permit, indeed to encourage, students to discuss their assignments with one another while they are working on them. A good ground rule is that any amount of discussion is allowable, but that students should write up their own solutions independently. Some mathematics departments set aside special "discussion rooms" where students are encouraged to get together and work on difficult problems.

On Correcting Papers

If a student has put forth a serious effort on a homework paper, you are obliged to read the paper carefully and to comment appropriately, either orally or by way of notes written on the papers before they are returned. A student who works hard and produces a well-constructed answer deserves more than a mere checkmark. Point out briefly where the work is especially good. If the solution is incorrect, identify the error or set the student on the right track. In some cases you might ask that a new solution be submitted. Of course, you may not be able to treat all the students in this ideal way all the time, but you can try.

It is well known that most problems can be correctly solved in various ways. If a student chooses a method different from the one you would have chosen, be certain that a "right answer" doesn't conceal fallacies or compensating errors. Even when the student's method is correct, it is not out of order to call attention to easier or more powerful methods when they exist. However, it should not be implied that a method is wrong just because there is a better one.

Sometimes problem solutions are posted or are reproduced and distributed to the students. This may be very helpful, but is not an adequate substitute for appropriate comments on the work of each individual student.

You undermine the student's developing responsibility and self respect, as well as respect for the subject, if you do not require that work be submitted on time in decently organized, legible form. Students should be required to write in complete sentences, with careful attention to logical connectives, and their writings should show steady progress toward the attainment of an acceptable mathematical style. This includes proper alignment of equal signs and fraction lines, using an equal sign where two quantities are equal and not using equal signs when they do not apply; for example, when asked for the derivative of $y = x^2$, the student should *not* be allowed to write $y = x^2 = 2x$ or some variant of this.

It should be made clear to the students that they have not really finished a homework assignment until they understand how to do the problems on it. One can reinforce this point by including on each test some problems taken directly from the homework.

Tests

> *"The whole art of teaching is only the art of awakening the natural curiosity of young minds for the purpose of satisfying it afterwards."*
>
> Anatole France,
> *The Crime of Sylvestre Bonnard*

Attitudes and Objectives

From the standpoint of student-instructor relationships and the cultivation of a cooperative team spirit, the usual examination system is a potential obstacle. Just when you have become established as the students' staunch ally, you are obliged to shift into the role of judge and jury and, it may be, executioner. At this stage particularly, you should seek means of preventing your own metamorphosis into a hatchetman in the eyes of your students.

It is essential that both you and the students be fully aware of the objectives of the examination and of the interpretation to be put on the results. A test is merely a quantitative observation of the process of learning. Both you and the student are being observed. The examination is an observation on your progress as a team.

The principal objectives of examinations are:

1. To evaluate each student's achievement.
2. To evaluate the instructor's teaching.
3. To provide a learning experience in the examination itself and in the review for it.

If the examination is to be a valid assessment of the student and the instructor, it must be very carefully formulated.

It is useful practice to select some test questions from homework assignments and from questions that caused difficulty in previous tests. This encourages students to keep previously covered topics fresh in their minds and not to sweep old difficulties under the rug. Of course, if this practice is used the students should be told!

There will be some situations in which you may call in a colleague to be the examiner. This method has the advantage of preserving the solidarity of the instructor-student team, but it has the disadvantage of placing a heavy responsibility on a guest examiner whose viewpoint, knowledge, and priorities may differ markedly from your own.

Examinations can be instruments for teaching even while they measure what students learn. A well-posed, invigorating test question may prompt students to do some serious thinking; even if they do not answer it successfully they may be eager to hear the answer when it is subsequently discussed in class.

If take-home examinations are feasible, the students can be led, especially in advanced courses, to carry out some important parts of the subject on their own. Material learned in this way is sometimes learned better than it is by other methods.

Types of Tests

Tests may be divided into two very broad and obvious classifications: *open-book* and *closed-book*. When students are permitted to use their text, notes, and reference material, the questions can be more authentically representative of problems met in practice. Memory for details—spontaneous recall—will play a secondary role in this type of test. The question is whether they can produce when they have access to the resources that would normally be at their disposal. This applies not only to the time-restricted open-book examination but also to the take-home examination. The take-home examination, for which a week or more may be allowed, tests the students' abilities to arrive at a solution at their own pace. However, the open-book examination is not always the prototype of professional practice. There are numerous professional situations in which the engineer, scientist, economist, or

mathematician must respond and act without the benefit of a library. Closed-book examinations are usually more appropriate in elementary courses since these courses consist primarily of basic mathematical skills which must be throughly mastered if the student is to progress satisfactorily to concepts and techniques of a more complicated nature.

Scheduling Tests

Individual preferences run from many short quizzes to a few comprehensive examinations. Frequent tests encourage students by reducing the course content to more manageable segments and by providing more nearly continuous assessment of progress. They also tend (as do frequent homework assignments) to even out students' study efforts over time. On the other hand, less frequent tests compel students to integrate larger segments of material in their minds and to retain the material over longer periods; they thus reward relative scholarly maturity. Also, they divert less time from other class activities. If asked to set its own examination schedule, a class will usually choose a reasonable plan well between the extremes.

In any case, it should not be possible for one bad test to ruin a student's grade for the course. The simplest protection against this is to schedule enough tests; another is to give, say, four tests and to count for each student only the best three, that is, throw out the lowest test score. Yet another is to replace the lowest test score with the final examination score if the latter is higher (the final examination in addition counting the usual way).

Give at least one test early in the term so that people who are in difficulty in your course will recognize their predicament and be able to do something about it before it is too late. Major examinations should be announced at least a week in advance.

Two principal advantages of a final examination are that it permits the student to integrate the material of the entire course and allows the instructor to assess the student's command of the course at its conclusion.

Composing Tests

Here are some suggestions to keep in mind when you sit down to write a test:

1. Begin by listing the *main* ideas, theorems and methods for which the student is held responsible, and try to cover most of them.

Don't despair if a few are missed, but it may be unfair to some students to accent one part of the material while completely neglecting others. The learning and review functions of a test should be given at least as much attention as the evaluation function.

2. Try to maintain a reasonable balance between "plug and grind" problems, applications, and theory.

Give an easy question or two first to allow less confident students to get off to a good start. The first few questions can be used to identify failing students by having those questions cover what you feel is absolutely minimum knowledge. Problems should range fairly uniformly from easy to difficult; otherwise all the students except the very best and the very worst may get grades that do not differ significantly from one another. For the same reason, it is wise not to weight problems by difficulty. If you do intend to assign different weights to questions, this information should appear on the examination.

For example, a test might be designed so that 40% are problems that can be solved by anyone who sat in class with open eyes, another 20% that can be solved by those who have done the homework routinely, 20% that are similar to harder homework problems, and finally 20% that are new (or a synthesis the students have not seen) and designed to indicate who the "A" students are.

3. Don't give students unnecessary difficulties (and excuses).

Be sure that the copy is readable and problems are clearly stated. Encourage questions during the examination about the interpretation of the problems, but reserve the right not to answer some questions. Try to avoid problems in which success depends too heavily on mastery of skills which it is not the purpose of the test to evaluate.

4. Making test questions significantly interdependent is a form of double jeopardy and should be avoided.

This is as important for the convenience of the grader as for fairness to the student. For example, if problem 1 consists of determining a function satisfying prescribed conditions, then problem 2 should not involve explicit use of that function, such as calculation of its derivative or integral. If a student arrives at an incorrect solution to problem 1, the use of this solution in problem 2 might lead to great difficulty. This could make the fair assignment of partial credit difficult and could adversely affect a student's performance on the remaining problems because of time limitations.

Similarly, repetition should be avoided. Each problem should have its own purpose, and no purpose is served by having two problems test the same concept.

5. Whenever possible, avoid the use of multiple choice tests.

Although the grading of such problems becomes quick and easy, it is extremely difficult to design such a test which will fairly evaluate the state of the student's progress. Students dislike them because they often do not offer sufficient opportunity to display the extent of their knowledge. Also a careless mistake may lead to the complete loss of points in a situation that would normally warrant substantial partial credit. Furthermore, questions of this form are not representative of the types of mathematical problems that arise in practice.

If the instructor finds that multiple choice tests are inevitable, it is still possible to give students the right to submit one or two solutions for partial credit on a separate sheet of paper. This eases student hostility to the multiple choice testing substantially, and makes the process fairer.

6. Don't ask for proofs unless students know exactly what they may assume.

Give "story problems". Make students apply definitions and work examples; avoid asking them for general definitions and theorems which merely test their ability to memorize, but not their mathematical progress. The important thing is whether the students understand and can use the general ideas, not whether these ideas have been learned by rote. Expect students to show their work and give reasons for what they do. Don't give full credit for unsupported correct answers unless the problem is trivial.

7. Before a test is used, work through it carefully, *using the actual form that the student will use* (or, preferably, have someone else do it).

It is unpleasant to discover afterward that a problem was defective, thus upsetting the students and making meaningful evaluation difficult.

In determining the length of a test remember that you have certain advantages over the student: general experience, previous exposure to the problems, and probably a more relaxed state of mind. A test that takes you more than fifteen minutes to do is almost certainly too long for a fifty-minute period.

The time by which examination papers must be handed in should be clearly announced and firmly observed. It is unfair to

allow some students to go on working when others must leave, for example to go to another class.

Grading Tests

1. Grade and return examination papers promptly.

Students conclude that their class has a low priority among your activities if more than one or two periods go by without their having the papers returned. The grades, your written comments on the papers, and classroom discussion of the examination will all mean more when the students' memories of the examination and of what they were thinking when they took it are still fresh.

2. Take reasonable steps to increase the objectivity and uniformity of grading.

Ask students to sign papers where you will not see their names every time you pick up the papers.

3. Decide in advance how you will allocate partial credit.

Certain common mistakes can be anticipated, and if penalties for these mistakes are predetermined, grading will be more consistent. Marking the mistakes on five or six papers will improve your ability to anticipate the errors you will enounter.

4. Grade one problem at a time.

Shuffle the papers between problems in order to offset possible tendencies to grade the first papers differently from later ones.

5. Do not grade by the answer alone; examine the technique carefully.

Mark wrong only something which is wrong. Watch for unusual and ingenious solutions; the wrong answer does not necessarily imply the method was incorrect.

6. When you return the papers, list the grade ranges and the median grade on the blackboard.

Without this information, a student with a low score may believe that almost no one in the class performed any better on the examination. If the class is small you may want to list all grades on the board. Perhaps letter grades corresponding to the scores should be mentioned—as tentative equivalents—to leave no ground for future complaints.

7. An examination often exposes ideas which are not yet clear to enough students.

Your last opportunity to dispel a particular misconception may come when the papers are returned, and the opportunity should not be lost. This is also a time when particularly elegant solutions by students may be shown to the class. If the examination covered some important concepts and perhaps unified several ideas, this is an opportunity to stress these matters again.

8. If time does not permit discussion of the examination at length, at least distribute a set of solutions when you return the papers.

Sets of solutions should not be distributed immediately after the examination is taken, however; this practice tends to cut off worthwhile discussion of the examination questions among the students themselves.

Good Endings

"To teach is to learn twice."

Joseph Joubert, *Pensées*

Proper planning before the course begins and minor adjustments during the term should avert any necessity for covering the second half of the course in the last week. The students profit little from attempts to cover material at breakneck speed toward the end of the term. More than likely, they will not be enthusiastic to begin another course in mathematics following one which does not have a satisfactory ending.

It is a revealing and sometimes embarrassing experiment to ask students to name the most important theorem they know. Even mathematics majors often come up with distressing answers. Not all of the blame for students' inability to judge the relative importance of ideas should be put on their shoulders. The end of the term is an ideal time for the instructor to give the students one last comprehensive view of the course, distinguishing the peaks from the hills and the hills from the plains—and explaining why the peaks are peaks. This also offers an excellent opportunity to suggest courses which the students might find appropriate to take after having completed the present course.

Assigning Grades

> *"A teacher affects eternity; he can never tell where his influence stops."*
>
> Henry Adams, *The Education of Henry Adams*

The assigning of grades at the end of the term is a task that deserves serious thought and considerable care. The final grades students receive in a course normally become part of their permanent records and may have far-reaching effects on their careers.

The instructor can prepare for this crucial phase of the term's work by constructing and grading tests with care and by keeping thorough and well-organized records of students' performances. The results of each examination and each set of homework should be clearly recorded, together with such information as the date, the class average, the weight it carries in the final average, and perhaps a brief note indicating what material was covered. Grade books are available for precisely this purpose. Besides facilitating the determination of valid final grades, care in recording can be a valuable aid in diagnosing problems of individual students along the way. It also enables a colleague to understand the basis for the grade you have assigned if some question arises when you cannot be reached. Should you be asked for a reference or evaluation of one of your students in the future, these records will serve to refresh your memory and document your opinions.

When final averages are computed according to the policy you have chosen, the resulting numbers will generally fall into reasonably well-defined groups with a few scattered intermediate cases. Once you have reached a decision about the clusters, the place where the cut-off points between the clusters should be chosen is best determined by looking at the records of the students falling between clusters. In determining these cut-off points you may wish to take into account such factors as participation in class discussions, whether the quality of work improved or declined during the term, evidences of understanding that do not show on the record, and so on. Make certain that students are aware of any unusual aspect of your grading policy, and do not modify grades using procedures you have not explained. For example, you may adopt a rule that, in your course, a student will always receive a grade which is no more than one grade below the one he or she has received in the final examination (thus assuring a student who has an A on the final examination at least a B in the course). Other deviations from the grading pattern after you have decided on the cut-off points should be strenuously avoided since they are likely

to make it very difficult, if not impossible, to convince a student of the justice of the assigned grade if he or she comes in to discuss it.

If there are several sections of the course you teach, be sure your grading is consistent with that of the other sections. Follow the grading policy established for the course. If there is only one section of your course, be sure that your grading is consistent with past grading policy for this course and the general grading policies of your department.

A student who questions the assigned grade deserves a fair hearing and a courteous review of his or her performance over the term. If your evaluation of the work has been based on careful deliberation, the student will usually be satisfied by such an analysis. Do not dismiss the student peremptorily, but consider the possibility that the student did in fact receive an inappropriate grade.

There are times when students will insist that they know the material, but for various reasons their records do not show it. At such times you should remind them that the letter grade reflects their actual performance and not what they might have done if . . .

II. FEEDBACK AND EVALUATION

"The decent docent doesn't doze:
He teaches standing on his toes.
His student dassn't doze—and does,
And that's what teaching is and was."

David McCord, *What Cheer*

All of us are evaluated by our students, whether we like it or not. The question is how these evaluations can be communicated in a timely and productive fashion. Open communication between faculty and students serves not merely to let the professor know whether students like the course, but also to make the students aware of the rationale behind the presentation. More importantly, it indicates to the instructor the areas in need of improvement.

Written evaluations made by students at the end of the term can be valuable to the instructor and to students in future courses, but they are of no help to students in the present course. More or less immediate oral evaluation or feedback, for instance in the form of questions asked, probably has much greater potential value. Instantaneous feedback, of course, need not even be oral; the perceptive teacher who looks at students while lecturing can hardly miss signs of puzzlement, boredom, or pleasure on their faces. If a perfect class, all the students understand everything that is being done all the time. Perfect classes do not exist, but a good teacher strives to approximate this ideal as closely as possible.

If, in spite of your standing invitation to ask questions, you sense that you have "lost" a student, you might pause and ask the student if something needs further explanation. But a word of caution: impatience on your part with the response may result in an impassive and unreadable face on that student for the rest of the term.

Another type of feedback that may be especially useful to the inexperienced teacher is obtained by spending the last few minutes of each period discussing what went wrong and what went right that day. LISTEN CAREFULLY to what the students have to say, even if it seems unreasonable. It might be a good idea to take notes. Feel free to tell the students how *you* feel about the way things went. If the class was dead, say so. Make it clear that the students share responsibility if the class is a drag.

Several times each term (more frequently near the beginning of the course) some time might be reserved when students are in-

vited to discuss more global aspects of the course, such as its pace, the usefulness of the text, quantity and quality of assignments, use of class time, the grading of homework, etc. Information of this sort can also be picked up by coming to the classroom early and chatting informally with your students. Since many students are afraid of making negative comments in class for fear of retribution, you might suggest that they take ten minutes of class time to give you a written evaluation of how the class is going. You can begin the next class session with a brief discussion of the comments, and what you've decided to do about them.

Depending on local conditions, more formal types of evaluation can occur in several ways.

1. TA's and other instructors often exchange visits to one another's classes.
2. In some institutions, equipment and operators are available for videotaping an instructor at work. This offers you an excellent opportunity to detect distracting mannerisms or weaknesses in your presentation.
3. You might invite a senior colleague who is respected as an excellent teacher to visit your class and afterward frankly discuss his or her reactions with you, and perhaps also note them down on a form made available for the purpose.
4. If you teach a large class, you may want to hold a meeting with your assistants at the end of the term to discuss and evaluate the course you have just finished.
5. You should consider asking all the students in your classes to fill out a questionnaire at the end of a course. Indeed, in many colleges and universities this has become standard procedure. In any case, although students have inevitable limitations as judges of the quality of teaching, the results of such surveys should be taken seriously and considered thoughtfully. If the questionnaire consists of numerical or other rating scales, the value of the survey is often enhanced if the students are encouraged to complement their ratings with comments. The usefulness of these responses will naturally be greater when measures have been taken to assure the students that they have nothing to fear by way of reprisal or exposure because of their comments.

III. READING AND SEMINARS RELATED TO TEACHING

To the extent that you are involved in it, teaching mathematics is your profession; it is therefore natural that you take a professional interest in it. This implies a continuing vital involvement with mathematics itself, but also leads naturally in various other directions: to the history of mathematics; to the philosophical, cultural, scientific, and even political environment of mathematics; to trends and new ideas in mathematics education; to the activities of certain professional organizations, and so on. More generally, it is useful to know something about educational theories and movements and about the history, philosophy, and current problems of higher education as a whole. It is possible, of course, to devote so much time to these matters that your development as a mathematician *and* your effectiveness as a teacher may suffer. A steady but controlled interest in them can nevertheless enhance your teaching not only by way of specific improvements but also by strengthening that intangible quality called perspective.

A simple way to cultivate this kind of professional interest is to browse through appropriate journals from time to time. These include:

The American Mathematical Monthly
The Two-Year College Mathematics Journal
Mathematics Magazine
The International Journal of Mathematical Education in Science and Technology

The first three journals are available through membership in the Mathematical Association of America.

It is easier to make a habit of scanning publications of these kinds if recent issues are kept in a separate, conveniently located collection—perhaps in a departmental lounge or in a corner of the departmental library. If your department does not already have such a collection, you might propose that one be established. The same collection might also contain a selection of suitable books. An extensive annotated bibliography on college and university teaching, containing over 250 entries, is available as part of *Change* Magazine's Undergraduate Teaching Project. Requests should be addressed to "Teaching Bibliography," *Change* Magazine, NBW