Geometry and Dynamics of Solenoids

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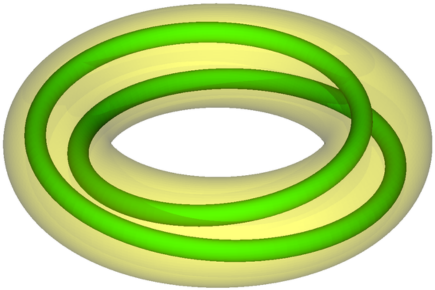
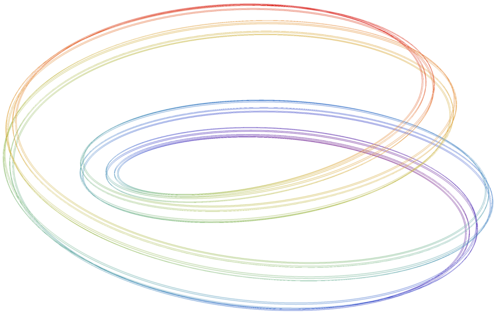
This is a story of how an example leads to new discoveries.

- The Smale/Vietoris solenoid, and its properties.
- Generalizations of (1-dimensional) solenoids
  (coverings of the Klein bottle)
- Modeling a solenoid by group actions on rooted trees
  (generalized odometers)
- Local invariants of odometers
  (non-Hausdorff elements)
- Applications to number theory
  (wild Galois groups)

Visualize an embedding of a solid torus into itself



After a few repetitions we get something like this



This is modeled by a sequence of coverings of the circle

$$\mathbb{S}^1 \xleftarrow{m_1} \mathbb{S}^1 \xleftarrow{m_2} \mathbb{S}^1 \xleftarrow{m_3} \mathbb{S}^1 \xleftarrow{m_4} \mathbb{S}^1 \xleftarrow{m_5} \cdots$$

- ⋆ each $m_\ell > 1$ is an integer
- ⋆ $m_\ell \colon \mathbb{S}^1 \to \mathbb{S}^1$ is an m_ℓ-fold covering map.

In the embedding construction, each $m_\ell = 2$, but no reason to restrict the degrees this way.

Repeat this covering construction infinitely often.
The inverse limit space is a Vietoris solenoid (1927)

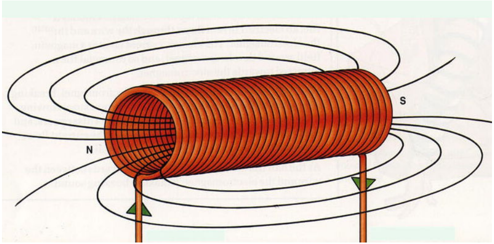
$$\begin{aligned}\mathcal{S} &\equiv \varprojlim \{\, m_{\ell+1} \colon \mathbb{S}^1 \to \mathbb{S}^1 \mid \ell \geq 0\} \\ &\equiv \{(x_0, x_1, \ldots) \in \mathcal{S} \mid m_{\ell+1}(x_{\ell+1}) = x_\ell \text{ for all } \ell \geq 0 \,\} \\ &\subset \prod_{\ell \geq 0} \mathbb{S}^1\end{aligned}$$

- ⋆ Give the product space the product or Tychonoff topology;
- ⋆ $\mathcal{S}$ has the restricted topology;
- ⋆ $\mathcal{S}$ is a compact, connected metric space, so a continuum.

Properties of the space $\mathcal{S}$

- ⋆ $\mathcal{S}$ is an uncountable union of lines, each dense in $\mathcal{S}$!

  $\mathcal{S}$ can be thought of as a bundle of wires, tightly coiled



Properties of the space $\mathcal{S}$

⋆ Whatever stage $k \geq 0$ you start at, the limit is the same:

$$\mathcal{S} \cong \varprojlim \{ m_{\ell+1} \colon \mathbb{S}^1 \to \mathbb{S}^1 \mid \ell \geq k \}$$

⋆ $\mathcal{S}$ is homogeneous - for any pair of points $x, y \in \mathcal{S}$ there is a homeomorphism $h \colon \mathcal{S} \to \mathcal{S}$ with $h(x) = y$.

⋆ Lastly, recall results of

- R.H. Bing, Canad. J. Math., 1960
- M.C. McCord, Transactions A.M.S, 1965

Theorem: Let $\vec{p} = \{p_1, p_2, p_3, \ldots\}$ and $\vec{q} = \{q_1, q_2, q_3, \ldots\}$ be sequences of prime numbers. Say that $\vec{p} \sim \vec{q}$ if there is $\ell_p, \ell_q \geq 0$ so that the two sets

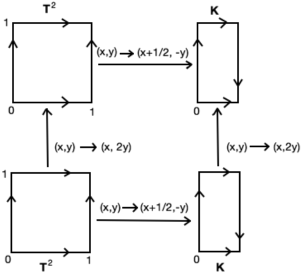
$$\{p_\ell \mid \ell \geq \ell_p\} = \{q_\ell \mid \ell \geq \ell_q\}$$

Then the solenoids they generate, call them $\mathcal{S}(\vec{p})$ and $\mathcal{S}(\vec{q})$, are homeomorphic if and only if $\vec{p} \sim \vec{q}$.

That is pretty much the end of the story - for 1-dimensional spaces!

Question: Which of these properties are valid, if we repeat this procedure for coverings of manifolds with dimension > 1?

Here is a nice example due to [Rogers & Tollefson, 1971/72], obtained by coverings of the Klein bottle.



Let $\mathbb{T}^2 = \mathbb{S}^1 \times \mathbb{S}^1$, and consider an involution

$$r \times i(x, y) = (x + \tfrac{1}{2}, -y).$$

The quotient $K = \mathbb{T}^2/(x, y) \sim r \times i(x, y)$ is the Klein bottle.

The double cover $L\colon \mathbb{T}^2 \to \mathbb{T}^2\colon (x, y) \mapsto (x, 2y)$ induces a double cover $p : K \to K$.

Define K_∞ to be the inverse limit of the iterations of $p : K \to K$.

Since $i \circ L = p \circ i$, there is a double cover $i_\infty : \mathbb{T}_\infty \to K_\infty$.

Space K_∞ cannot be homogeneous, as there is exactly one leaf with $\mathbb{Z}/2\mathbb{Z}$ as isotropy group for the monodromy action.

Let M_0 be a connected closed manifold, and let $f^\ell_{\ell+1} : M_{\ell+1} \to M_\ell$ be a sequence of finite-to-one proper covering maps.

Compose these coverings to get

$$f^0_{\ell+1} \colon M_{\ell+1} \to M_0$$

Pick basepoints $x_{\ell+1} \in M_{\ell+1}$ with $x_\ell = f^\ell_{\ell+1}(x_{\ell+1})$. Then

$$G_\ell = \mathrm{Image}\left\{(f^0_{\ell+1})_\# \colon \pi_1(M_{\ell+1}, x_{\ell+1}) \to \pi_1(M_0, x_0) \equiv G_0\right\}$$

is the subgroup for which $M_{\ell+1}$ is the associated covering space.

Basic Fact: The Galois action on the fibers $(f^0_{\ell+1})^{-1}(x_0)$ is transitive iff $G_{\ell+1}$ is a normal subgroup of G_0.

- ⋆ For G_0 abelian, every subgroup is normal.
- ⋆ For G_0 not abelian, "normality" becomes a central question.

Now take the next step, and form the inverse limit space:

$$\begin{aligned} M_\infty &= \varprojlim \; \{f^{\ell}_{\ell+1} \colon M_{\ell+1} \to M_\ell \mid \ell \geq 0\} \\ &= \{(y_0, y_1, y_2, \ldots) \mid f^{\ell}_{\ell+1}(y_{\ell+1}) = y_\ell \mid \ell \geq 0\} \\ &\subset \prod_{\ell \geq 0} M_\ell \end{aligned}$$

is a compact connected metrizable space called a *(weak) solenoid*.

For each $\ell > 0$, there is a fibration map $\Pi_\ell \colon M_\infty \to M_\ell$.

For fixed $x_\ell \in M_\ell$ the fiber $\mathfrak{X}_\ell = \Pi_\ell^{-1}(x_\ell) \subset \mathfrak{X}_0$ is a Cantor space.

The path connected components of M_∞ are manifolds, which are non-compact covering spaces for M_0, so that M_∞ is a generalized lamination, or foliated space with Cantor transversals.

The fundamental group $G_0 = \pi_1(M_0, x_0)$ acts on the fiber $\mathfrak{X}_0$ via lifts of paths in M_0 to the leaves of $\mathcal{F}_{\mathfrak{M}}$ giving the monodromy action on the fiber, $\Phi\colon G_0 \times \mathfrak{X}_0 \to \mathfrak{X}_0$. The action is minimal, the orbit of each point is dense in $\mathfrak{X}_0$.

The monodromy is an *equicontinuous group action*:

- A Cantor action $\varphi\colon G \times \mathfrak{X} \to \mathfrak{X}$ is equicontinuous if for some metric $d_{\mathfrak{X}}$ on $\mathfrak{X}$, for every $\epsilon > 0$ there exists $\delta > 0$ such that

$$d_{\mathfrak{X}}(x,y) < \delta \implies d_{\mathfrak{X}}(\varphi(g)(x), \varphi(g)(y)) < \epsilon \quad \text{for all } g \in G.$$

When $G = \mathbb{Z}$ then a minimal equicontinuous Cantor action $\varphi\colon G \times \mathfrak{X} \to \mathfrak{X}$ is conjugate to a classical odometer.

When the action of G is free - there are no fixed points for the action of any $g \in G$ except the identity - then the action is called a (strong) odometer in the works

- [Cortez & Petite, *J. London Math Soc.*, 2008]
- [Cortez & Medynets, *J. London Math Soc.*, 2016]

Our interest is in the general case, when there is no assumption that the action is free, such as for the coverings of the Klein bottle, and other even more interesting constructions. These actions are also called *subodometers* by Cortez & Petite.

Two dimensional laminations formed by branched covers of $\mathbb{C}$ in works of Lyubich & Minsky, and Nekrashevych, are not exactly the same as weak solenoids, but their monodromy group actions are.

Research Focus:

⋆ What can one say about the geometry of the leaves of the foliation $\mathcal{F}_{\mathfrak{M}}$ of M_∞ ?

⋆ When is the space M_∞ homogeneous?

⋆ Give criteria sufficient to imply that two weak solenoids M_∞ and N_∞ are homeomorphic.

In a Cantor group action $\varphi\colon G \times \mathfrak{X} \to \mathfrak{X}$, a clopen subset $V \subset X$ is *adapted* if the restricted action to V is that of a subgroup $G_V \subset G$.

Definition: Minimal equicontinuous group actions $\varphi_0, \varphi_1\colon G \times \mathfrak{X} \to \mathfrak{X}$ are *return equivalent* if there exist adapted clopen subsets $V, U \subset \mathfrak{X}$, and subgroups G_V, H_U such that the restricted actions $\varphi_0\colon G_V \times V \to V$ and $\varphi_1\colon G_U \times U \to U$ are conjugate.

Theorem: If two weak solenoids are homeomorphic then their monodromy actions are return equivalent.

In fact, with stronger assumptions, one gets:

- Return equivalent $\Longleftrightarrow$ homeomorphic weak solenoids.

Summary:

- Smale solenoids are simplest example of 1-dimensional solenoids

- Essentially all questions about 1-dim solenoids are answered.

- Weak solenoids form huge class of generalizations.

- Classification of weak solenoids leads to study of minimal equicontinuous Cantor actions.

- Need to understand return equivalence for minimal equicontinuous Cantor actions.

- Need more techniques to answer questions for weak solenoids.

Profinite model for a Cantor action is powerful, limited usefulness.

Definition: The closure $E(\Phi)$ of $H_\Phi = \Phi(G) \subset \mathbf{Homeo}(\mathfrak{X}_\infty)$, in the topology of pointwise convergence on maps, is called the *Ellis (enveloping) semigroup*.

Proposition: Let Φ be an equicontinuous Cantor action. Then $E(\Phi) = \overline{H_\Phi}$ = closure of H_Φ in the *uniform topology on maps*.

For $x \in \mathfrak{X}_0$ let $\mathcal{D}_x = \{h \in \overline{H_\Phi} \mid h(x) = x\}$ be its isotropy group.

Lemma: The left action of $\overline{H_\Phi}$ on $\mathfrak{X}_\infty$ is transitive, hence $\mathfrak{X}_\infty \cong \overline{H_\Phi}/\mathcal{D}_x$. This is the *profinite model* for $\mathfrak{X}$.

The closed subgroup $\mathcal{D}_x \subset \overline{H_\Phi}$ is independent of the choice of basepoint x, up to topological isomorphism.

Theorem: If $\mathcal{D}_x$ is trivial, then M_∞ is homogeneous.

$\mathcal{D}_x \cap \Phi(G) = \{id\}$ is possible with $\mathcal{D}_x \neq \{id\}$.

Group chain model for a Cantor action is clumsy, but very useful.

Given a descending chain of proper subgroups of finite index

$$\mathcal{G} \equiv \{G_0 \supset G_1 \supset \ldots \supset G_\ell \supset \cdots\}$$

we obtain a Cantor set $G_\infty = \varprojlim \,\{G/G_\ell \to G/G_{\ell-1}\}$, with a minimal left G-action $\Phi_0 \colon G \times G_\infty \to G_\infty$.

Theorem: [folklore] Given a minimal equicontinuous action $\varphi \colon G \times \mathfrak{X} \to \mathfrak{X}$, there exists a group chain $\mathcal{G}$ such that the action $\Phi_0 \colon G \times G_\infty \to G_\infty$ is conjugate to φ.

Relating the profinite and group chain models:

$C_\ell = \bigcap_{g \in G} g G_\ell g^{-1} \subset G_\ell$ is the normal core of G_ℓ in G.

C_ℓ has finite index in G, so define the profinite group

$$C_\infty \equiv \varprojlim \; \{q_{\ell+1} \colon G/C_{\ell+1} \to G/C_\ell \mid \ell > 0\} \; .$$

Each group G/C_ℓ acts on the finite set $X_\ell = G/G_\ell$, so there is an induced action $\widehat{\Phi}_\infty \colon C_\infty \to \mathbf{Homeo}(C_\infty)$.

Theorem [Dyer-Hurder-Lukina, 2016]. $\overline{H_\Phi} = \widehat{\Phi}_\infty(G_\infty)$, and

$$\mathcal{D}_\infty \equiv \varprojlim \; \{\pi_{\ell+1} \colon G_{\ell+1}/C_{\ell+1} \to G_\ell/C_\ell \mid \ell \geq 0\} \; \cong \mathcal{D}_x \; .$$

The fundamental group of the Klein bottle is the dihedral group

$$G_0 = \pi_1(K, 0) = \langle a, b \mid bab = a^{-1}, b^2 = 1 \rangle.$$

For the cover $p^\ell \colon K \to K$, $\ell \geq 1$,

$$G_1 = p_* \pi_1(K, 0) = \langle a^2, b \rangle \ , \ \ G_\ell = (p^\ell)_* \pi_1(K, 0) = \langle a^{2^\ell}, b \rangle$$

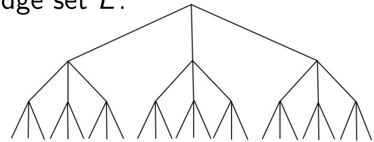
The cosets of G/G_ℓ are represented by $a^i G_\ell$, $i = 0, \ldots, 2^\ell - 1$,

$$C_\ell = \bigcap_{g \in G} g G_\ell g^{-1} = \langle a^{2^\ell} \rangle \ .$$

Then $G_\ell / C_\ell = \{C_\ell, bC_\ell\}$, and so $\mathcal{D}_x \cong \mathbb{Z}/2\mathbb{Z}$.

Tree model for a group chain action is geometric.

Let $\mathcal{T}$ be a tree with vertex set V and edge set E:



$V_0 = \{v_0\}$ is root vertex at level 0.

The vertices at level ℓ given by $V_\ell = G_0/G_\ell$.

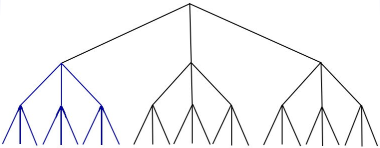
Edges between V_ℓ and $V_{\ell+1}$ correspond to elements of $G_\ell/G_{\ell+1}$.

Let $\mathcal{P}(\mathcal{T})$ be the space of semi-infinite rays in $\mathcal{T}$.

Give $\mathcal{P}(\mathcal{T})$ the cylinder topology, then it is a Cantor set.

G acts on $\mathcal{T}$ by permuting vertices at each level so that the paths in $\mathcal{P}(\mathcal{T})$ are preserved. The arboreal action $\Phi \colon G \times \mathcal{P}(\mathcal{T}) \to \mathcal{P}(\mathcal{T})$ is conjugate to the group model action $\Phi_0 \colon G \times G_\infty \to G_\infty$.

Return equivalence for arboreal actions:



$\Phi\colon G \times \mathcal{P}(\mathcal{T}) \to \mathcal{P}(\mathcal{T})$

Let $x = (v_k) \in V$ be a vertex point.

Let $U(v_k) \subset \mathcal{P}(\mathcal{T})$ be the set of all paths through v_k.

Then $U(v_k)$ is a basic clopen subset of $\mathcal{P}(\mathcal{T})$.

The restriction of the action of G to $U(v_k)$ is given by the action of the subgroup $G_k \subset G$, so have $\Phi_k\colon G_k \to \mathbf{Homeo}(U_k)$.

For each $\ell \geq k$, the normal core of G_ℓ in G_k is

$$C_{k,\ell} = \bigcap_{g \in G_k} g G_\ell g^{-1}. \quad \text{Observe that } C_{k,\ell} \supset C_\ell.$$

The isotropy group of the action of $\overline{\Phi(G_k)}$ at $x \in \mathcal{P}(\mathcal{T})$

$$\mathcal{D}_{x,k} = \varprojlim \; \{G_\ell / C_{k,\ell} \to G_{\ell-1}/C_{k,\ell-1} \mid \ell \geq k\}$$

is the *discriminant group* of the action Φ_k.

We have coset inclusions $G_i/C_i \to G_i/C_{k,i}$.

Theorem 1: [Dyer-Hurder-Lukina, 2017] Let $\Phi_0 \colon G \times G_\infty \to G_\infty$ be the Cantor group action for the group chain $\{G_i\}_{i \geq 0}$, let $x \in G_\infty$ be a point. Then for any $k > j \geq 0$ there is a well-defined surjective homomorphism $\Lambda^{k,j} : \mathcal{D}_{x,j} \to \mathcal{D}_{x,k}$.

The action is called stable, if there exists j_0, such that for all $k > j \geq j_0$ the homomorphism $\Lambda_{k,j}$ is an isomorphism.

If such j_0 does not exist, then the action is called wild.

Definition: [Hurder-Lukina, 2017] The *asymptotic discriminant* of $\Phi_0 \colon G \times G_\infty \to G_\infty$ is the equivalence class (with respect to tail equivalence) of the chain of surjective group homomorphisms

$$\mathcal{D}_{x,0} \to \mathcal{D}_{x,1} \to \mathcal{D}_{x,2} \to \cdots .$$

The asymptotic discriminant is an invariant of Cantor actions.

Theorem: [Hurder-Lukina, 2017] The asymptotic discriminant of a Cantor group action $\Phi_0 \colon G \times G_\infty \to G_\infty$ is invariant under the return equivalence of actions.

In the tree model, choose a ray to infinity, $x \in \mathcal{P}(\mathcal{T})$, and form the chain of discriminant groups for the isotropy action at each vertex along the ray. This is a *germinal* invariant of the action.

1. All equicontinuous actions of abelian groups on Cantor sets are stable, with trivial discriminant group $\mathcal{D}_x$.
2. If a discriminant is finite, then it is stable. The action of the dihedral group defined earlier is stable with discriminant $\mathbb{Z}_2$.
3. Every finite group and every separable profinite group can be realized as the stable discriminant group of an action of a torsion-free finite index subgroup of $\mathbf{SL}(n, \mathbb{Z})$ on a Cantor set.
4. There exists uncountably many wild actions of torsion-free finite index subgroups of $\mathbf{SL}(n, \mathbb{Z})$ with distinct asymptotic discriminants.

The proof of 3 and 4 uses ideas of **Lubotzky 1993** on torsion elements in the profinite completion of torsion free subgroups of $\mathbf{SL}(n, \mathbb{Z})$, and a construction due to **Lenstra**.

Theorem: [folklore] Let G be a finitely-generated group, and $\varphi\colon G \times \mathfrak{X} \to \mathfrak{X}$ be an equicontinuous minimal Cantor action. Then there is a weak solenoid M_∞ whose monodromy action is conjugate to φ. Moreover, this can be achieved for a 2-dimensional solenoid, whose base manifold M_0 is a closed surface.

Corollary: Every group listed in the previous Theorem is realized as the discriminant invariant for a weak solenoid.

- Upon the passage from 1-dimensional solenoids to higher dimensional solenoids, you enter a wild world of possible behaviors for the dynamics of their monodromy actions.

Summary:

- Three models for minimal equicontinuous Cantor actions:
- profinite model defines isotropy/discriminant group of action
- group chain model calculates discriminants
- arboreal model suggest local invariants at ends of branches
- all three models yield constructions of examples
- Need to understand meaning of these invariants

There is a geometric interpretation of the stable/wild property.

The notion of LQA actions was introduced by **Álvarez López and Candel 2009** as a generalization to topological spaces of the notion of quasi-analytic actions on manifolds due to **Haefliger**.

Definition: A topological action $\Phi \colon G \times \mathfrak{X} \to \mathfrak{X}$ is *locally quasi-analytic (LQA)* if there exists $\epsilon > 0$ such that for any open set $U \subset \mathfrak{X}$ with $diam(U) < \epsilon$, and for any open $V \subset U$ and $g_1, g_2 \in G$ if

$$\text{if } \Phi(g_1)|V = \Phi(g_2)|V \text{ then } \Phi(g_1)|U = \Phi(g_2)|U \ .$$

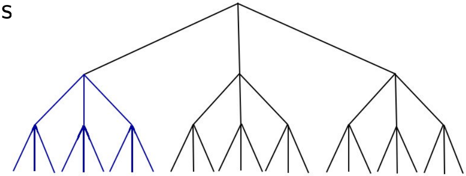
Alternatively, the action is locally quasi-analytic if and only if for all $g \in G$ if $\Phi(g)|V = id$, then $\Phi(g)|U = id$, for open sets $V \subset U$.

Theorem: [Hurder-Lukina, 2017] A minimal equicontinuous action $\Phi \colon G \times \mathfrak{X} \to \mathfrak{X}$ is stable if and only if the action of the Ellis group $\overline{\Phi} \colon \overline{\Phi(G)} \times \mathfrak{X} \to \mathfrak{X}$ is locally quasi-analytic.

Example: Let $(\mathcal{T}, v_0)$ be a rooted tree. Then the action of $\mathbf{Aut}(\mathcal{T}, v_0)$ on $\mathcal{P}(\mathcal{T})$ is not LQA.

Let $\mathcal{T}_d$ be a d-ary tree, then $\mathbf{Aut}(\mathcal{T}_d, v_0) = S_d^{\infty}$, where S_d is the permutation group on d elements, and S_d^{∞} denotes the infinite wreath product.

In the wreath product, the actions on subtrees are independent, so the restricted action on $U(v_k)$ has multiple extensions to $U(v_{k-1})$.



Let $\mathfrak{X}$ be a Cantor set, and let $G \subset \mathbf{Homeo}(\mathfrak{X})$ be a group.

An element $g \in G$ is *non-Hausdorff* if there exists $x \in \mathfrak{X}$ and a collection of open sets $\{U_n\}_{n \geq 1}$ with $\bigcap U_n = \{x\}$, such that

1. $g(x) = x$,
2. $g|_{U_n}$ is non-trivial,
3. for $n \geq 1$, there exists an open set $O_n \subset U_n$ such that $g|_{O_n} = id$.

Theorem: [Winkelnkemper, 1983] The germinal étale groupoid $\Gamma(\mathfrak{X}, G)$ associated to the action of G on $\mathfrak{X}$ is non-Hausdorff if and only if G contains a non-Hausdorff element.

Lemma: [Hurder-Lukina, 2018] Let $G \subset \mathbf{Homeo}(\mathfrak{X})$ be a (countable or profinite) group acting minimally on a Cantor set $\mathfrak{X}$. Suppose $g \in G$ is non-Hausdorff, then the action of G is non-LQA.

Question: If the action of $G \subset \mathbf{Homeo}(\mathfrak{X})$ is non-LQA, does it have to contain a non-Hausdorff element?

The answer is negative when G is a countable group.

This example, due to Lukina, was found amongst the absolute Galois groups for function fields, and their arboreal representations.

Summary:

- Classification of weak solenoids leads to study of minimal equicontinuous Cantor actions.
- Asymptotic discriminant is invariant of return equivalence.
- Stable actions satisfy continuous orbit equivalence rigidity.
- Wild actions may also have non-Hausdorff elements.
- Wild actions give rise to new classes of C^*-algebras.
- Applications to absolute Galois groups.

The analogy between theory of finite coverings and Galois theory of finite field extensions suggests looking for examples of minimal Cantor actions arising from purely arithmetic constructions.

• [R.W.K. Odoni, 1985] began the study of arboreal representations of absolute Galois groups on the rooted trees formed by the solutions of iterated polynomial equations.

• [Rafe Jones, 2013] gives an introduction and survey of this program, from the point of view of arithmetic dynamical systems and number theory.

Here is a typical result from the recent work

- O. Lukina, *Galois Groups and Cantor actions*, arXiv:1809.08475

Theorem: [Lukina, 2018]

Let $f(x)$ be a quadratic polynomial with integer coefficients. Let P_c be the orbit of the critical point c of $f(x)$, and suppose P_c is finite. Let $Gal_{\rm geom}(f)$ be the geometric iterated monodromy group, that is, the Galois group of the infinite field extension of $\mathbb{C}(t)$ obtained by adjoining to $\mathbb{C}(t)$ the roots of $f^n(x) - t$, for $n \geq 1$.

If $\# P_c \geq 3$, and the post-critical orbit is pre-periodic, the asymptotic discriminant of the action of $\mathrm{Gal}_{\rm geom}(f)$ is wild, and $\mathrm{Gal}_{\rm geom}(f)$ contains *non-Hausdorff elements*.

More is true, as Lukina realizes essentially all of the phenomena discussed in this talk, as the dynamical behavior for arboreal representations of Galois groups of function fields.

Problem: Develop relations between invariants for minimal equicontinuous Cantor actions that arise from arboreal representations of Galois groups of function fields and number fields, and properties such as the density of prime divisors for sequences such as $\{f^n(\alpha)\}_{n\geq 0}$ for an algebraic integer α.

Welcome to the wild world of solenoids!

Welcome to the wild world of solenoids!

Thank you for your attention.

References

R.H. Bing, *A simple closed curve is the only homogeneous bounded plane continuum that contains an arc*, **Canad. J. Math.**, 12:209–230, 1960.

N. Boston and R. Jones, *Arboreal Galois representations*, **Geom. Ded.**, 124, 2007.

A. Clark and S. Hurder, *Homogeneous matchbox manifolds*, **Trans. A.M.S.**, 365:3151–3191, 2013.

J. Dyer, S. Hurder and O. Lukina, *Molino theory for matchbox manifolds*, **Pac. Jour. Math.**, Vol. 289:91-151, 2017.

R. Fokkink and L. Oversteegen, *Homogeneous weak solenoids*, **Trans. A.M.S**, 354(9):3743–3755, 2002.

S. Hurder and O. Lukina, *Wild solenoids*, **Trans. A.M.S.**, to appear, 2018, arXiv:1702.03032.

R. Jones, *Galois representations from pre-image trees: an arboreal survey*, in **Actes de la Conférence "Théorie des Nombres et Applications"**, 107-136, 2013.

A. Lubotzky, *Torsion in profinite completions of torsion-free groups*, **Quart. J. Math.** 44:327–332, 1993.

O. Lukina, *Arboreal Cantor actions*, **Jour. L.M.S.**, to appear, arXiv:1801.01440.

O. Lukina, *Galois Groups and Cantor actions*, preprint, arXiv:1809.08475.

M.C. McCord, *Inverse limit sequences with covering maps*, **Trans. Amer. Math. Soc.**, 114:197–209, 1965

R.W.K. Odoni, *The Galois theory of iterates and composites of polynomials*, **Proc. L.M.S.**, 51:385-414, 1985.

R. Schori, *Inverse limits and homogeneity*, **Transactions A.M.S.**, 124:533–539, 1966.

L. Vietoris, *Über den höheren Zusammenhang kompakter Räume und eine Klasse von zusammenhangstreuen Abbildungen*, **Math. Ann.**, 97:454–472, 1927.