Pockets

Teacher Lab Discussion

Overview and Introduction to Frequency Distributions

"How do I know that I am correct?" is a question all scientists face and fear. If we make a measurement, we want to be sure that the measurement is right. If we ask a question, we want to have confidence that our answer will stand up. We do not want to be embarrassed by someone else giving a different answer and then finding they were correct and we were wrong. Accuracy is vital to all areas of science.

So, how do we ensure accuracy? We make repeated measurements of the same thing. We do not drop a ball once from a height of 40 cm; we drop it 3 or 4 times. Each partner reads the graduated cylinder or checks the balances. In most instances that is all that is necessary—just a few trials, a couple of independent checks, and then the "best" or "most likely" value is picked and used as *the* result.

Most of the time a few trials are all that is necessary because you can control most of the variables in the experiment. When you drop a tennis ball, it is the same tennis ball dropped each time onto the same floor, and you always drop it the same way. That consistency, that control of extraneous variables, is crucial to minimizing the variability of the results of a single measurement. Still the results do vary a little bit. Drop a tennis ball from a height of 40 cm and it may bounce to 20 cm, or 22 cm, or 19 cm. It may hit a crack in the floor, it may have a worn or soft spot, you might not release it from exactly 40 cm, and it is hard to measure the height of a bouncing ball. All of these variations contribute small errors, so you carry out three trials.

Sometimes, however, scientists deal with situations in nature where the variability is very great and one cannot do anything about it. In these situations even three measurements would be far too few to tell us what we want to learn. For example, if I want to know how many beans I can pull out of a container with my hand, one pull is not likely to give me the answer. Only by pulling several times can I get an idea of how many beans I might be able to pull out in any one grab. Fishermen netting salmon might want to know how many salmon they will get in one day so that they can figure out their costs and profits. Only by keeping track of the data for many days can they get a reasonable idea of what they expect their daily catch to be. We may need 20 or 30 measurements to answer our question, and then, all we may be able to say is "well, the result is *most likely* 5 beans," or, in the case of the fishermen, "the *probability* of getting 5 tons of fish is 20%." This is not as satisfying as saying "the result is 5," but it is the best we can do. So be it.

Besides repeated measurements of a single quantity, scientists and mathematicians often want to measure a single quantity for a large number of objects. For example, if we are interested in the height of adults in the United States, it wouldn't do to measure one person 50 times. It would be far more informative to measure the height of 50 different people once. Then, at least, these 50 measurements would give us a representation of the heights of adults in the USA. Our representation would be more realistic if we could measure 1000 people, or even 10,000. But that is not practical. So we measure as many people as we can and say things like, "the *most likely* height is ...," or "30% of the people are taller than ..."

In either case, lots of measurements of something becomes an experiment in and of itself. Any time you make 20 measurements of anything, you have the right to call it an experiment. We give this type of experiment a special name. We call them frequency distribution experiments, and these experiments use the ideas of probability to analyze the results. Probability is a mathematical topic as important as addition, subtraction, and geometry. For that reason, starting in first grade we shall use frequency distribution experiments to learn about probability, and we shall do at least one frequency distribution experiment in each subsequent grade.

In a typical frequency distribution experiment we measure how often (i.e., how frequently) certain numbers keep coming up. Our first frequency distribution experiment is called *Pockets* and was developed by Ms. Mary Empfield for her 1st grade class at the Andrew Jackson School in Chicago.

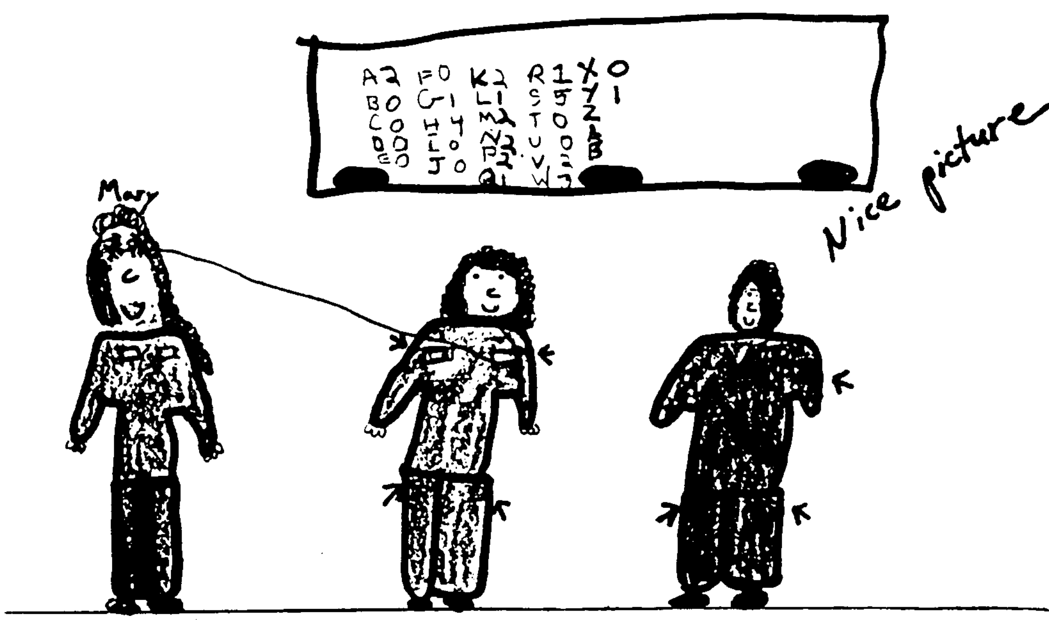
Every child could have pockets. But what is the number of pockets a child has? Depending on what the child wears, we might get any number from 0 to 10 or more pockets for our answer. Indeed, there is no one answer for a class of thirty children. But, by doing a frequency distribution experiment, we certainly can find the most likely number of pockets for the class. We can find the range of pockets, which is the largest number of pockets minus the smallest number of pockets. Also, we can make some educated prediction of how the most likely number of pockets and the range might change as we change some of the variables in the experiment. So, we are on our way to learning about probability.

The number of pockets is the manipulated variable. Let's call this variable P. P may range from no pockets, i.e., P = 0, to possibly 10 or more pockets. How many children have a particular number of pockets is the responding variable. Let us call the number of children C. The value of C is determined by the experiment, that is, by counting how many children have 0 pockets, how many have 1 pocket, 2 pockets, etc. For each value of the manipulated variable, P, there will be a corresponding value of the responding variable, C.

Picture, Data Table, and Graph

As always, we start with a picture of the experiment. We ask the children to use arrows to help indicate what each symbol means. Thus, they should draw an arrow from the letter P to the pockets of each child in their picture. The letter C could point to each child in the picture, since C tells us how many

Figure 1



children have a certain number of pockets. Kiela's picture from Ms. Empfield's class is shown in Figure 1. Notice the pockets are clearly shown with little arrows pointing to each of them. One of the arrows should have a P next to it. Also, a C should have been in the picture pointing to each of the children. Kiela put her teacher, Ms. Empfield, in the picture and shows her teacher clearly keeping an eye on everything. And, as Ms. Empfield indicated, it is a nice picture. A little praise goes a long way.

In Questions 1 and 2, we deal with the idea of the manipulated and responding variable but we do not use these terms yet. We start with the sentence, "Variables have values" and then state in **Question 1** that, "I know ahead of time the values of: (Circle one)." The children have the choice of circling:

P		*C*
number of pockets classmates might have.	or	*number of classmates with 5 pockets.*

In **Question 2**, we substitute, "I do not know ahead of time..." and then continue with the same choices. This is not an easy question for 1st graders to answer, but they have had practice in the previous grab bag experiment. Still, you might want to lead a class discussion on how to think about the question. You might start by asking if they know ahead of time how many pockets Billy (name a student) might have. Some children may say 2, others may say 3 or 4 depending upon the time of year. "Aha," you say. "Do we know ahead of time the values of P?" And the answer would be yes. Next ask, "Do you know how many in the class have 3 pockets before we do the experiment?" The answer should be no. At this point you can read the first two questions to the children and let them answer them.

The variables which do not change values are the focus of **Question 3**. You can ask them if the time of year might make a difference. Also ask how their results might change if they did the experiment on the beach instead of at school. Certainly there will be more pockets in the winter than in the summer, when long-sleeved shirts and sweaters replace T-shirts. As for the beach, hardly anyone has pockets on the beach. Let each child come up with an idea of his or her own. Some ideas may be off the wall and very silly, but some will be very good. These good ones are the beginning of critical thinking. If you can get the children to appreciate that the time of year and the place are controlled variables, then you have done a great job.

Each child should count his or her own pockets (all of them, shirt or blouse and pants or skirts) and write the results under his or her name on the blackboard. Then each child records the names and P in Table I of their lab write-up. One should learn always to keep the raw data so that if there are difficulties with the data, one can check what one is doing. An example of Table I is shown in Figure 2.

Figure 2

Table I: Raw Data Table

	Name	P Number of Pockets
1.	*Bill C.*	*5*
2.	*Andy C.*	*3*
3.	*Barb C.*	*4*
4.	*Howard G.*	*5*
5.	*Shandra F.*	*6*
6.	*Jose J.*	*5*
7.	*Annie P.*	*0*
8.	⋮	⋮

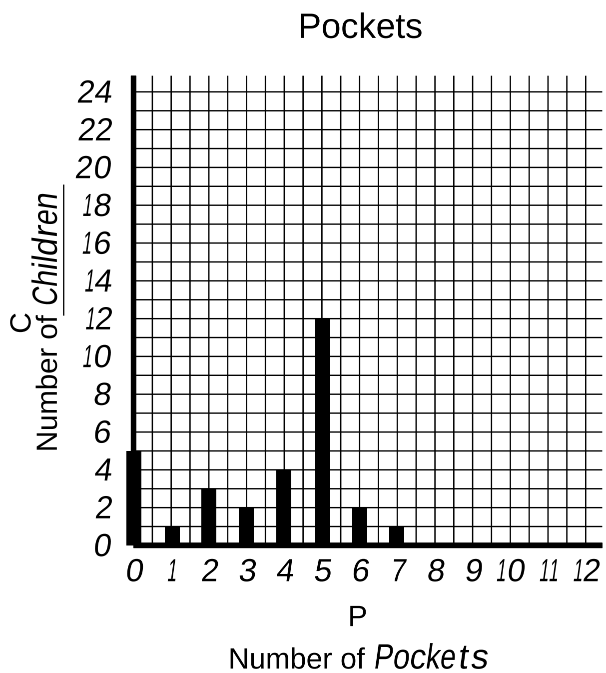
Once this information has been recorded, each child turns the raw data into a frequency distribution using Table II. The best way to do this is to first put a tally mark next to a particular value of P each time it occurs. So, for Bill C. a tally mark goes next to P = 5, then for Andy C. a mark next to P = 3, and for Barb C. one next to P = 4. When Howard's name is reached, a second tally mark is placed next to P = 5, etc. If the names in Table I are checked off as the corresponding numbers of pockets are tallied in Table II, then the children are less likely to lose track. Group cooperation will be useful here. One child might go through the names

Figure 3

Table II

P Number of *Pockets*	C Number of *Children*	
	Tallies	Total
0	卌	5
1	I	1
2	III	3
3	II	2
4	IIII	4
5	卌 卌 II	12
6	II	2
7	I	1
8		0

Figure 4



in Table I, saying the name and how many pockets, while another child makes tallies in Table II.

The children must also complete the labeling of Table II. Next to the letter P, the children should fill in what P stands for, i.e., number of pockets. Next to the letter C should be what C stands for, i.e., number of children. An example of a frequency distribution table is shown in Figure 3.

When all the tally marks are in Table II, they are added and the final total, C, for each value of P is recorded. This is a nice experiment to do when the children are learning about addition. Counting by fives also comes into play. All the tallies should add up to the number of children in the class. Indeed, each tally mark represents one child. In our example there were 30 children in the class.

As always, we want to make a graph of the data. This time we give the children a little help with the numbering. The rest is up to them. Since there are no in-between values of P, like 4.5 pockets, a bar graph is appropriate. **Question 4** and **Question 5** ask which axis is used to "put" the number of pockets and which to "put" the number of children. The number of pockets is put on the horizontal axis and their occurrence, C, is put on the vertical axis. The graph of the frequency distribution data is shown in Figure 4. The data is from Ms. Empfield's class. Note the five children with no pockets. Could these be girls with skirts and pullover tops?

Comprehension Questions

The comprehension questions have two main goals. One is to introduce the children to the idea of probability through the wide range of the number of pockets in the experiment that cluster around a "most likely" value. The second is for the children to exercise their math skills and to learn math outside the context of the textbook.

Question 6 and **Question 7** focus on the numbers for the most and least likely occurrence of pockets. For our sample data, the most likely number is 5 pockets, and the least likely number is both 1 and 7 pockets. Checking on the children's ability to read the graph or data table, we ask in **Question 8** how many classmates have 3 pockets. Our sample answer is 2 children. That's not very many. "There's only a small chance that someone will have 2 pockets" is an expression you should use with the children. "What is the likelihood...?" "what is the chance...?" "what is the probability...?" are all expressions we want the children to become familiar with in an operational way. *Pockets* is one way to do this.

We combine reading the graph, doing addition, and the ideas of greater than or less than in Questions 9 and 10. The children are asked to make two calculations, one to find out how many children have 0, 1, 2, or 3 pockets in **Question 9**, and one to find how many have 4 or more pockets in **Question 10**. Adding, we have the number of children with 0, 1, 2, or 3 pockets:

5 + 1 + 3 + 2
= 6 + 5 (counting on twice)
= 11 children,

while the number of children with 4 or more pockets is

4 + 12 + 2 + 1
= 16 + 3 (counting on twice)
= 19 children.

We have used some mathematical "tricks" so that the children can add (or subtract) a string of numbers without using a calculator. In particular, one adds small groups of numbers using techniques like counting on, so that one gets a few larger numbers. One then uses counting on again to get the final answer. Sometimes doubling comes in handy, say, when 5 + 5 appears. We shall indicate what "trick" we are using in our calculations usually by parenthetical statements, as we have done above.

In **Question 11**, we ask, How many children did this experiment? The quickest way to answer the question is to realize that adding the previous two answers, 11 + 19 = 30 children, will give the correct answer. If the children do not see this, then they will have to add up all the values of C all over again.

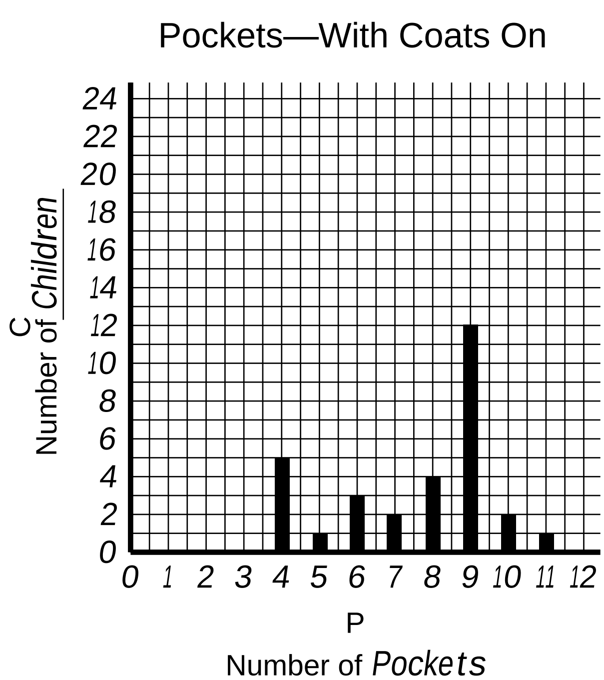
We are back to probability in **Question 12**, asking the children to predict the likelihood of a new child in class having 0, 1, 2, or 3, or 4, or more pockets. The new child is likely to have 4 pockets or more because 19 children had 4 or more, while only 11 had fewer than 4. But, of course, no one should be surprised if the new child has fewer than 4 pockets. Anything is possible, only some cases are more likely than others.

We continue to explore the distribution of pockets in Questions 13 and 14. In **Question 13** we ask how many of the classmates had the most pockets. That would be one who had 7 pockets. In **Question 14** we ask how many had the fewest. In our case 5 children had no pockets whatsoever.

In Question 15 we work on the idea of the spread, or range, of the number of pockets, which is the difference between the most pockets and the fewest anyone had. Moving along the horizontal axis, the children should see that the most anyone had was P = 7 pockets (**Question 15a**), the fewest P = 0 pockets (**Question 15b**), and the range of pockets (**Question 15c**) was

Range = 7 pockets – 0 pockets
= 7 pockets.

Figure 5



In **Question 16**, we look at the idea of addition in math. That is, you can take a result that you already know and add on (or subtract off) new information without doing the whole experiment all over again. For Question 16 we have our coats on and each coat has 4 pockets. What is the new frequency distribution? You take each value of P and add on 4 pockets. The values of C stay the same, but they are now for P + 4 pockets. Thus, the data of C = 5 children at P = 0 pockets now becomes the data C = 5 children at P = 4 pockets, etc. If needed, there is a Data Table III for them to record their original data with coats off. Some may be able to go directly to Table IV for coats on. The most likely number has moved up to 9. If the children do not see that from the data table, they should be able to when they plot the results of Table IV in **Question 16b**. Our results for the graph are shown in Figure 5.

Questions 17 through **22** can serve several purposes. You can give them to children who finish early or they can be used as homework problems. Or you might want to save them as quiz questions or use them as a model for quiz questions. The six questions are all directed at reading and interpreting a graph. The children must identify the most likely number of pockets, the largest and smallest number of pockets, the range of pockets, and the total number of children in the class. They are also asked to speculate, with an explanation of their reasoning, if the data was collected in winter or in summer. The most likely number of pockets is 7 pockets (**Question 17**). The largest number is 9 pockets (**Question 18**), the smallest number is 3 pockets (**Question 19**). This then leads to our definition of range (**Question 20**). The range is

$$\text{Range} = 9 - 3$$
$$= 6 \text{ pockets.}$$

With the most likely number of pockets so high, one would deduce that the experiment must have been done in the winter (**Question 21**). No T-shirts here!

We have saved the hardest question, **Question 22**, for last. One has to add up all the values of C to find the total number of children in the class.

$$\text{Total} = 1 + 5 + 3 + 8 + 12 + 6 + 2$$

One can try grouping numbers first by counting on twice:

$$= 9 + 8 + 12 + 8$$

then try doubling,

$$= 9 + 12 + 16$$

then count on twice again,

$$= 21 + 16$$
$$= 37 \text{ children.}$$

Remember that the experiment should take several periods. First, *lead* a discussion on the ideas of probability and what variables are involved. Then have the children draw a picture, collect the data, make the graphs, and finally answer the questions. Let the children compare graphs to see if they are correct, and let them figure out what is wrong before you intercede.

Summary

Probability is a way of life and so are pockets. *Pockets* is a good way to start on the road to understanding more about both. The essential point in establishing the probability that something will occur hinges on taking repeated measurements of the event. By counting and graphing the number of pockets each child has, we develop a sense of the frequency of a given number of pockets. This type of knowledge allows us to predict future events with some degree of certainty. It also helps paint a larger picture, from which we can infer the conditions in which the data were gathered. The thought process behind prediction and inference are integral to much of our understanding of the world around us.

Materials per Team

- No special materials are required for this lab.