Full of Beans

Teacher Lab Discussion

Overview

This is a simple experiment that deals with several fundamental concepts in an informal way. Major ideas include the notion of volume, counting and grouping in tens, addition and subtraction, bar graphing, variables, and controlled variables.

The children work in groups of two or three. Each group has a cup and two types of beans. The questions we will ask are, "Will the cup hold more of one kind of bean than the other?" and "How much more will it hold?" The first question is a qualitative comparison; the second is quantitative. One of the goals of TIMS is to progress, as scientists do, from qualitative questions to quantitative questions.

We would like the number of beans to be large enough so that grouping by 10's has to be used to count, but not so large that your students become frustrated. TIMS teachers have had success using 2-oz. plastic cups (the kind used for dressing at salad bars), kidney beans, and large lima beans. These cups hold approximately 40 lima beans and 80 kidney beans.

Picture, Data Table, and Graph

A picture of the experiment is shown in Figure 1. We show a full cup of beans and define two types of beans. The latter are labeled with a letter T for type of bean. One of the students has the right idea, "Let's group them by 10's!" The letter N indicates the number of beans in the cup to be counted.

In **Question 1**, we ask the students to write what the two variables are. They should indicate type of bean and number of bean by writing them out or using the symbols T and N. **Question 2** asks what remains the same during the experiment. You should have the children discuss this question. Do they realize why it would be a bad idea to change to a different size cup during the experiment? Once the experiment is over, we could do a new experiment with a different cup—but that's a different experiment.

Before the children actually collect the data, we ask (**Question 3**) which variable they can fill in before doing the experiment. In this, as in many other experiments, the two variables play somewhat different roles. One variable has *values* that are known before we do the experiment. In this experiment we know the values of T. We can fill in lima and kidney beans before we carry out the experiment. This variable is called the *manipulated*

Figure 1

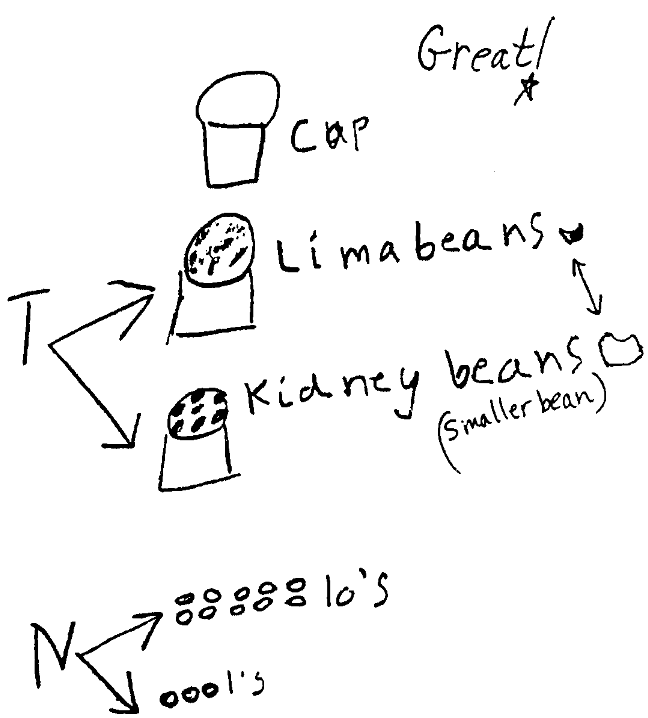
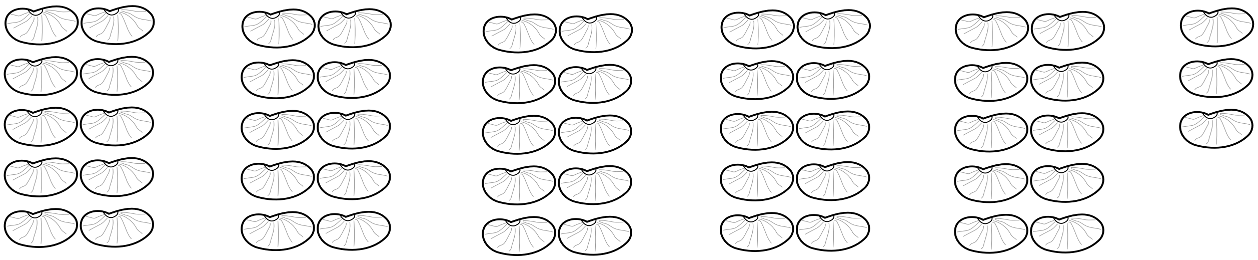


Figure 2

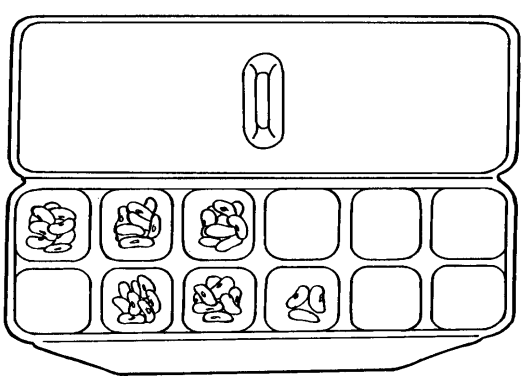


53 Lima Beans

variable. We decided ahead of time to use lima beans and kidney beans. You might decide on some other types of beans or maybe something else, like pasta. The values of the other variable, the numbers of beans, are not known until we do the experiment. You could point out that this is called the *responding variable*.

To collect the data, the children will have to fill the cup with beans. Of course there is a slight problem: When exactly is the cup full? They will have to use some judgment. We want the top of the beans to be just about level with the top of the cup. The children should jiggle the cup a bit so the beans settle down.

Figure 3



Once the cup is full of beans and level with the top, the beans have to be counted. Without some help, the children will most likely lose count and get quite frustrated. How do we avoid frustration? This is a time when we see the power of grouping by tens. Ideally, we would like the children to count out groups of tens and organize them neatly, as shown in Figure 2. Unfortunately, this has never worked in a real classroom. Children are never this neat. One solution we have found is to use egg cartons. Each egg "cell" can easily hold 10 beans (Figure 3). The only drawback of using egg cartons is that you cannot readily tell if your students have made a mistake and put 9 or 11 beans into one of the cells.

As with the experiment *Rolling Along with Links*, this is an activity that does not result in one precise answer. To average out differences in the way the cups are filled, we ask each partner to fill the cup

Figure 4

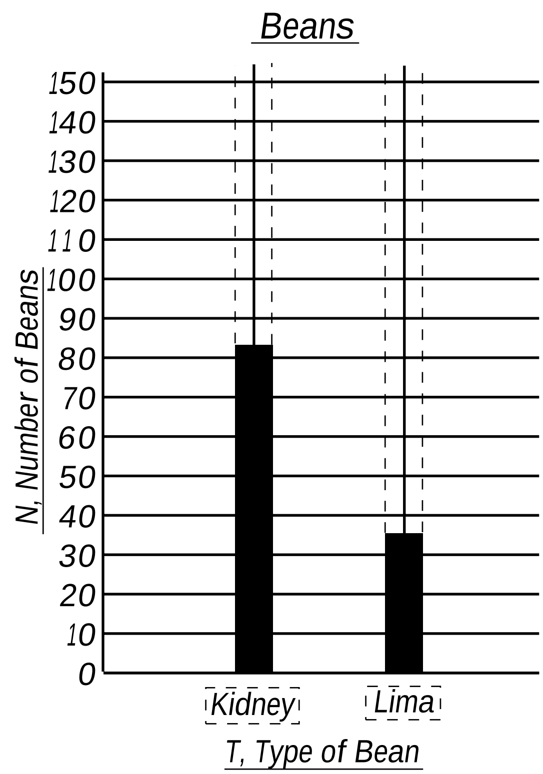
Table I

T Type of Bean	N Number of *beans*			
	Trial 1	Trial 2	Trial 3	Average
Kidney	*83*	*84*	*80*	*83*
Lima	*37*	*36*	*34*	*36*

separately and as best they can. The partners should cooperate in the process of counting and then enter the answer under Trial 1, Trial 2, or Trial 3. What if, as is most likely, the different trials do not result in the same number of beans? We do not want to introduce any formal definition of average at this time, such as adding the three trials and dividing by 3, since they are too young for that. So, once again, we use the eyeball average (see the *TIMS Tutor: Averages*). Our completed data table, using the 2-oz. cup, is shown in Figure 4.

The next step in our experimental process is drawing the graph. By now your students should be fairly proficient in bar graphing. The only new wrinkle in this experiment is that the numbers are large and therefore they cannot number the vertical axis by ones or twos. We have provided a graph on which the vertical axis is already numbered by tens. That means the children will have to figure out how to draw a bar to represent a number like 53. At this age we cannot expect too much accuracy. Certainly we would like them to realize that 53 is between 50 and 60. Ideally, we would hope they would mark the top of the bar closer to 50 than to 60. Before drawing the bars, the students label each axis with its proper variable. We have put a reminder in the form of a line. Once this is done each bar should be labeled with the kind of beans: lima, kidney, etc. The graph that goes with the data table in Figure 4 is shown in Figure 5.

Figure 5



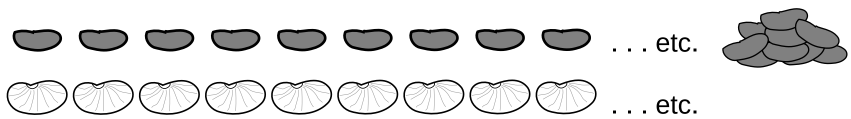
Comprehension Questions

Now we get to the heart of the experiment, the questions. In **Question 4**, we ask of which kind of bean did the cup hold more. In our example, the cup held more kidney beans. The next question, **Question 5**, is a comparison-type subtraction problem. We want to know how many more kidney beans (83) than lima beans (36) fit in the cup. This is a difficult question for first grade. At this point, the children do not know the subtraction algorithm, so we really have a problem-solving situation where they have to come up with a strategy. They may not even recognize this as a subtraction problem! Any correct strategy the children devise is perfectly acceptable. For example, they could use counters, such as Unifix cubes, they could draw a picture of the two collections of beans, or they could use the actual beans. One strategy is simply to pair up the two kinds of beans as much as possible and then to count how many of the small ones are left over (see Figure 6).

A problem with this strategy is that they are likely to lose count. Here again, we hope the children think of using the Unifix cubes and grouping by tens. This is a lot less messy. Undoubtedly, some groups will think of this idea and soon the others will follow suit. An example of using Unifix cubes to make the comparison is shown in Figure 7. They should discuss the strategies they use to find how many more Unifix cubes are in the kidney bean group.

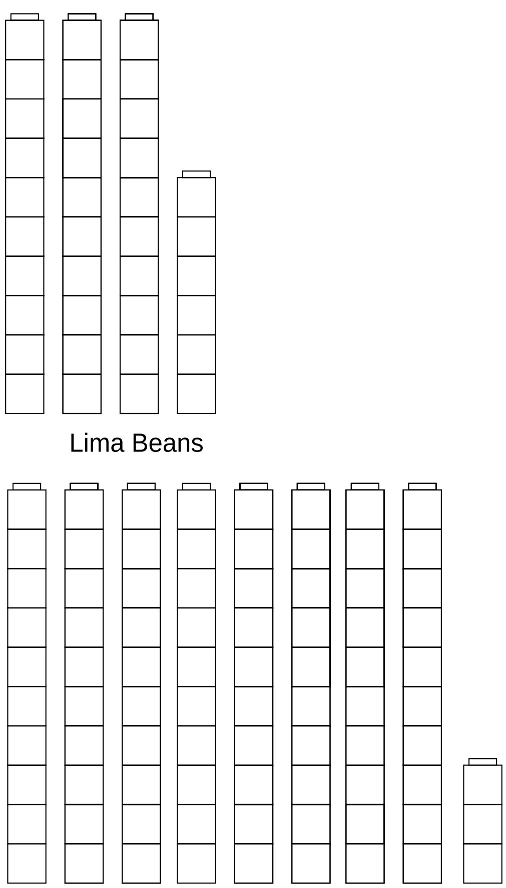
In Question 6 and Question 7 we want to get the children to think about the relationship between the size of the bean and the number that are

Figure 6



required to fill the cup. In **Question 6**, we ask which kind of bean is bigger. The children can easily see by direct comparison that the limas are bigger than the kidneys. This is also borne out by the fact that the cup holds fewer limas than kidneys. When they are much older, we will be able to put this in quantitative terms and note that we get an inverse relation. At this point we stick to qualitative ideas and see if they can realize that a cup always holds more small beans than big ones (**Question 7**). In fact, this is a rather subtle idea for first graders and you will be interested to see the variety of explanations they give.

Figure 7



Question 8 deals with another important concept. Another group doing the same experiment may get different numbers. Your students look at another team's result and see if the numbers are the same. Almost certainly the answer to **Question 8a** will be no. The "why" question we ask in **Question 8b** is a bit subtle and should generate some good class discussion. What was different in the way the two groups did the experiment? The cups used are exactly the same, so that can't account for the difference. On the other hand, the beans are not exactly the same, and the way they fit in the cup is not exactly the same. There could also be some differences in judgment in the way the two groups labeled a cup as "full." Finally, there is always the possibility that one of the groups made a mistake in counting.

In **Question 8c**, we ask which group was correct. This is very important to the TIMS philosophy. When doing many types of experiments there is not just one right answer. Both groups could be right, even though they got different answers. Does that mean they can never be wrong? No! If the experiment is carried out properly, the answers both groups get should be close. The problem is, what do we mean by close? When the children are older, we can define close in terms of percentages. (We'll start by having answers within 10% of each other be close.) For first graders, you will have to tell them what close means. For the large lima beans, 10% is 3 or 4, so just tell them that a difference of 3 or 4 is okay. For the kidney beans, 8 to 9 is close.

The remaining questions explore arithmetic and fractions in the context of beans. In **Question 9**, André filled one cup with 35 pinto beans. How many beans fill two cups? We want to get across the idea of doubling. Two cups should hold twice as much as one cup. Since there are 35 beans in one, the total number in both should be

35 + 35 = 70 beans.

The children can solve this by using Unifix cubes or counting up by 10's from 35: "45, 55, 65" and then adding 5 to give 70. Alternatively, they can set out beans in groups of 10 and 5 and then count them all.

In **Question 10**, Victor's and Mary's jars held 29 and 41 beans, respectively. Who had more? Clearly, Mary. How many more? The number sentence we are looking for in Question 10 is

41 – 29 = 12 beans.

Although we see this as a subtraction problem, most children may see it as an addition problem. Mary had 12 more beans than Victor, so

29 + 12 = 41 beans.

This is also a good answer. Ultimately, we would like the children to realize that the addition sentence

29 + 12 = 41 beans,

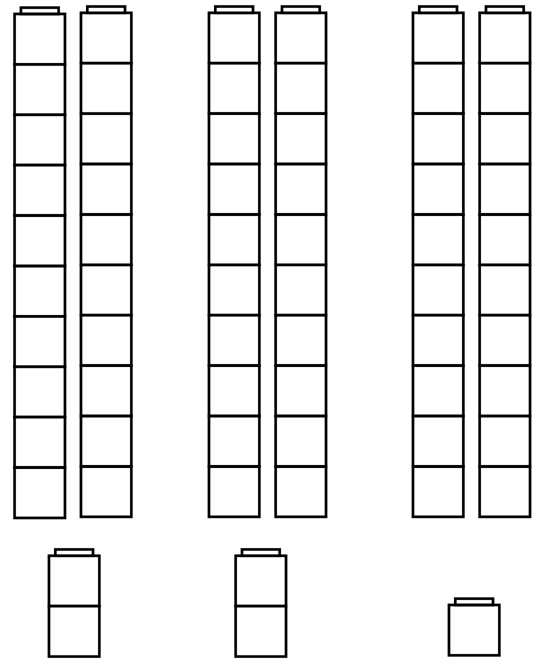
and the subtraction sentence

41 – 29 = 12 beans,

are really two sides of the same coin.

In **Question 11**, we have a division problem. Michelle filled 3 cups with 65 lima beans. About how many are in one cup? Again, using manipulatives (beans, Hex-A-Links, Unifix cubes, etc.), the children will have to take 65 objects and separate them into three equal groups. You will note that we chose a number that is not evenly divisible by 3. This is intentional. The question only asks for an estimate ("*about* how many lima beans?"), so acceptable answers include "21," "22," and "21 or 22." Observe the strategy the children use to solve the problem. Using Hex-A-Links or Unifix cubes and grouping by tens make the solution easier. When the class discusses the solution to this problem, see if any of the children first split the six tens into three groups of twenty and then divided up the ones. One example of a first grade solution is shown in Figure 8.

Figure 8



Question 12 is an exercise in graph reading. If you like, you can use it as an assessment or homework. For **Question 12a**, the exact answer is 43 beans. For first grade children we would be happy with anything in the range of 41 to 45 beans. Of course, the closer they get to 43, the better. In **Question 12b**, we get a bit tricky when we ask which bean is bigger. The bar for the kidney beans is bigger. This will lead many children to leap to the conclusion that the kidney beans are bigger. The correct answer requires at least two steps of reasoning. If the kidney beans have a bigger bar, then more of them fit in the cup. So they are smaller and the pinto beans are bigger.

In **Question 13**, Vonette has 17 beans, and she wants 25. How many more does she need? Question 13 is a subtraction problem that can be done either by counting or by using manipulatives. Some acceptable number sentences are:

subtraction: 25 – 17 = 8 beans,

or counting on: 17 + 8 = 25 beans,

where again the answer is N = 8 beans.

Question 14 comes back to the concept of division once more. Francis has 45 beans. About half are white, and about half are brown. How many brown beans are there? As in Question 11, we only ask for an estimate. Either "22," "23," or "22 or 23" are acceptable answers. A discussion with the class should bring out that the third form is preferable, since there is no way to decide between 22 or 23. You may have some students also come up with an exact answer of $22\frac{1}{2}$. In fact, you will find that some of the beans do split in half, but we are just interested in an estimate.

Question 15a is a "join" type addition problem. Again, using manipulatives, a calculator, or some other method, they should find 74 beans altogether. The third part of the question is more difficult. There are 31 pinto beans. Is this more than half of all the beans? We are back to multistep logic. The children may realize that 18 + 25 is more than 31 so less than half the beans are pinto. On the other hand, they may use their manipulatives to find half of 74, namely 37. Again, we hope they will find a variety of strategies and compare them with their neighbors' strategies.

Summary

To summarize, this seemingly simple experiment, which only requires filling a cup with some beans, explores a variety of mathematical and scientific concepts that remain important throughout the children's school career and beyond.

Materials per Team

- one 3-oz. cup
- two types of beans of different size, sufficient to fill the 3-oz. cup