Spreading Out I

Teacher Lab Discussion

Overview

This experiment is a way to introduce to children in the lower grades the "counting sq cm" technique of finding area. By dropping colored water on several brands of paper towel, the children create random-shaped spots of moisture. They then use a 1-cm grid to determine the areas of the spots, eventually graphing and analyzing their data. The brand of towel is the manipulated variable and the area is the responding variable.

Picture, Data Table, and Graph

A picture of the experiment done by Angela is shown in Figure 1. As you can see, the *values* of the manipulated variable—Scott, Bounty, and Heritage—are clearly identified, as is the eyedropper. The teacher in the school wisely picked three brands of paper towels that were widely different-priced paper towels and, therefore, of different quality. This assures us that the spot areas will be of different sizes.

Questions 1, 2, 3, and 4 focus on the variables. The type of paper towel is identified as the manipulated variable for **Question 1**, with the values being the brand name (**Question 2**). In answer to **Question 3**, the area of the spot produced by 3 drops of water is the responding variable. **Question 4** asks about the fixed variables. In this experiment the type of eyedropper and the number of drops used are held fixed.

Figure 1

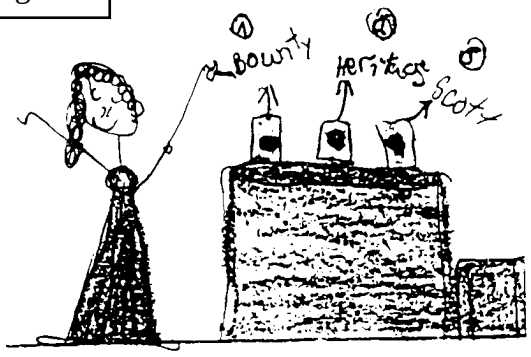


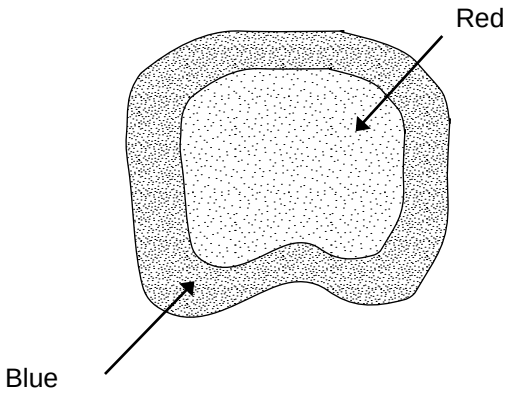
Figure 2

Table I

T Type of Towel	A Area in $\frac{\text{sq cm}}{\text{units}}$			
	Trial 1	Trial 2	Trial 3	Eyeball Average <A>
Bounty	$8\frac{1}{2}$	$7\frac{1}{2}$	$8\frac{1}{2}$	$8\frac{1}{2}$
Heritage House	19	$18\frac{1}{2}$	20	19
Scott	$9\frac{1}{2}$	$9\frac{1}{2}$	$8\frac{1}{2}$	$9\frac{1}{2}$

One does not have to use three drops on each towel. Two or four will do, but it is important that the children are careful and put the same number on each towel. Enough drops should be used to give a decent-sized area, but we don't want to spend all day counting square centimeters; hence the choice of three. It is a good idea to prop up each towel so that the center does not touch the table. This keeps the water on the towel and off the table. A book under each side of the towel will accomplish this nicely. Each partner should carry out a trial on each towel and determine the area. Then the two partners should collaborate on a third trial and together determine the area. All the data should be recorded and a final eyeball average found, as shown in Figure 2.

Figure 3



If the area of the spot is about 10 sq cm on Bounty, you can expect variation of about 1 sq cm between partners. Although water works quite well, a mixture of 30 drops of red food coloring and 10 drops of blue food coloring in 100 cc of water produces a lovely two-tone spot illustrated in Figure 3. As soon as the spot has stopped spreading, have the children trace out the perimeter of the spots using pencils as shown in Figure 4.

Once the perimeters are traced, the children must carefully cut out the spots and trace them onto a piece of sq-cm graph paper, labeling the area as

Figure 4

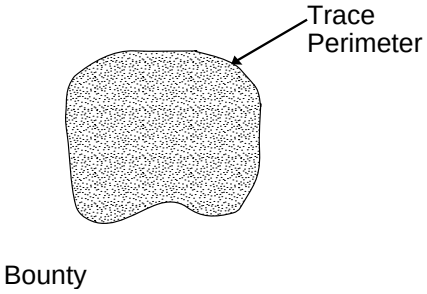
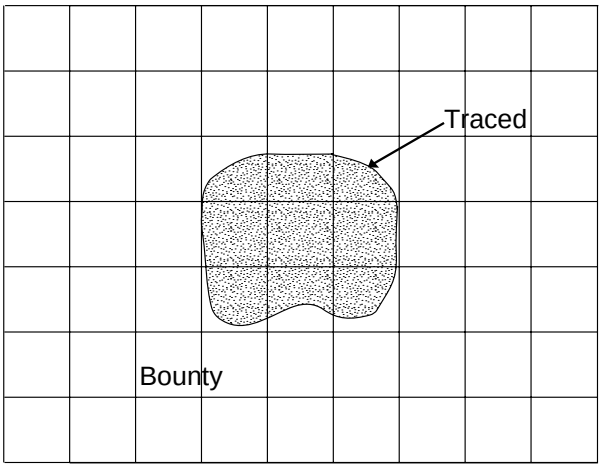
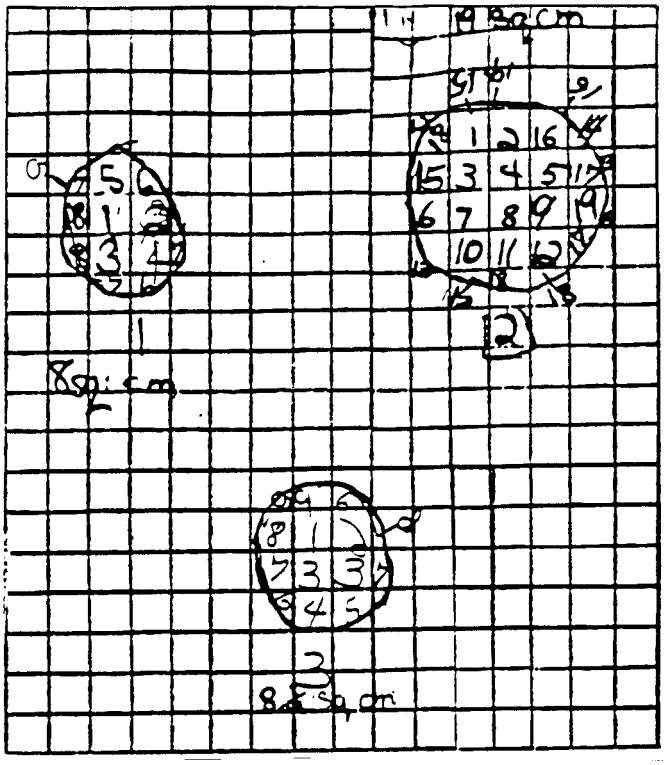


Figure 5



shown in Figure 5. Angela's own grid and three areas are shown in Figure 6, along with her technique for finding A. (Notice Angela's error in one of the spots.) This technique for finding area

Figure 6



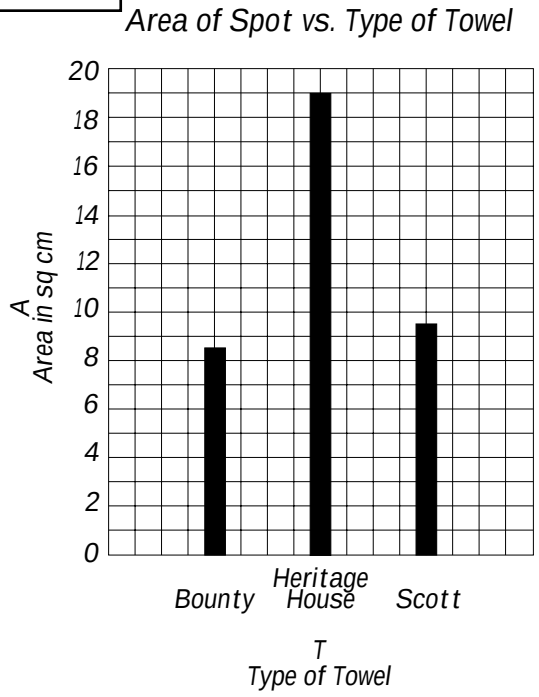
has been discussed in the *TIMS Tutor 4. Concept of Area,* so we will not repeat it here. The graph of the data is shown in Figure 7.

At this point a few questions on graphing are in order. To answer **Question 5**, about using a bar graph, the children should explain that towel type is a qualitative variable. **Question 6** asks where this variable should be placed on the graph. Qualitative variables are ordinarily placed on the horizontal axis. **Question 7** investigates towel type even further, asking if any one type must appear first on the axis. Since no particular data pattern is expected here, there is no necessary order.

Comprehension Questions

First we set up three questions on the children's own work. These questions test their ability to read and interpret bar graphs. The answers will, of course, be different if the children do not use the same brands of towels as Angela. **Question 8** and **Question 9** ask the children to identify the largest and smallest spot areas. For Angela's experiment, Heritage House had the largest spot area and Bounty had the smallest.

Figure 7



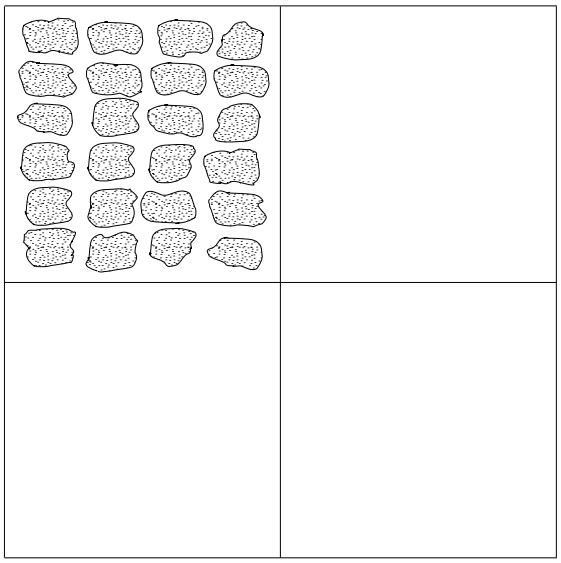
Question 10 is a bit trickier. It asks the children to imagine what would happen if they had dropped twice as many drops. The shape of the graph should look the same. The bars, however, will be higher (qualitative answer), and in fact each should be twice as high (quantitative answer). There is no *a priori* reason to expect that Angela will know that the area is twice as great because we explore the relationship between area and the number of drops in *Spreading Out II*.

In **Question 11** we give you a chance to see if the children can apply their results to a slightly different situation. Questions 11a and 11b ask the children to read data from a graph. In **Question 11a**, the Ace towel has the biggest spot, and in **Question 11b**, the spot on the generic towel has an area of 10 sq cm. Since the children work in pairs, it is always a good idea to ask a question that each child will have to answer independently, so that you can tell if both children understand the experiment.

In **Question 12** we want to determine if the children understand the basic technique of counting square cm to find area. They are asked to relate the graphed data in Question 11 to an unlabeled tracing of a spot on a grid. They must determine the area of the spot and then decide which type of towel it is by referring to the graph. The area is about 6 sq cm, which is close enough that the school towel is the only possible correct answer.

In **Question 13** we find out how many drops it takes to cover each towel. How do the children find the number of drops without doing this experiment? The children can obtain an answer to Question 13 by moving the spot around and making a series of tracings. Clearly, the smaller the spot area the more of them it will take to cover the towel. Since most towels are about 28 cm × 28 cm (11″ × 11″), the children will have to make an awful lot of tracings. We can cut this down to a more manageable problem and also do some simple math by dividing the towels into quarters. The children can find out how many spots it takes to fill a quarter towel, and then either multiply by four or add the number four times. This is illustrated in

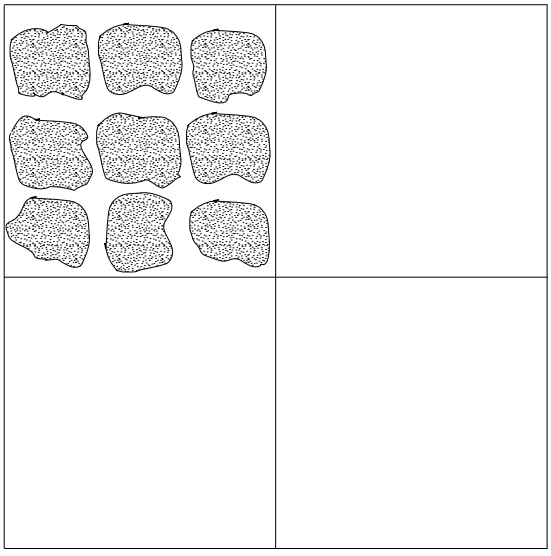
Figure 8



Bounty

Figure 8 for the Bounty towel and Figure 9 for the Heritage House towel. Some of the children might be super clever and "see" the situation shown in Figure 10 for the Bounty towel. By tracing out just 9 spots in an ordered array and then using multiplication, they can quickly determine that each quarter towel can hold 24 spots. This is much less time consuming than drawing 24 spots and also brings out the power and beauty of using mathematics.

Figure 9

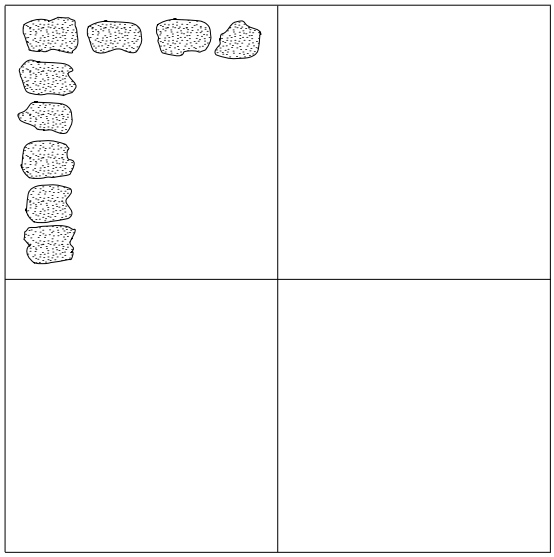


Heritage House

Using Angela's data, we managed to fit in about 24 spots on a quarter of the Bounty towel and only 9 spots on the Heritage House towel. To find out how many spots it would take to cover a full towel, we multiply by four, which for Heritage House is 36 spots. The number of drops picked up by a full towel is then the drops per spot times the number of spots or, again, for Heritage House,

$$D = 36 \times 3 = 108 \text{ drops.}$$

Figure 10



Bounty

Notice that our answer is approximate since there are some open places on each towel that we did not cover with a tracing. This causes an error in our estimate, but that error gets smaller as the area of a single spot decreases. If the children are not multiplying, then they can use repeated addition. For Bounty the number of spots on the towel is:

$$S = 24 + 24 + 24 + 24$$

which equals:

$$S = 20 + 4 + 20 + 4 + 20 + 4 + 20 + 4$$

$$= 80 + 16$$

$$= 96 \text{ spots}$$

and the number of drops is:

$$D = 96 + 96 + 96$$

$$= 288 \text{ drops}$$

These are big numbers for the children—but by second grade they should be able to handle up to 1,000. Of course, there is always a calculator! This question is a lovely exercise in determining area properly, estimating, and using simple multiplication or repeated addition.

Question 14 is tricky. It asks which of your towels will wipe up the most water. Many children will say that the towels with the biggest spot will wipe up the most water. You know, bigger is better. But as Question 13 just illustrated, the biggest spot for our data (19 sq cm) belonged to Heritage House and could absorb only 108 drops of water. The smallest spot ($8\frac{1}{2}$ sq cm) belonged to Bounty and it could absorb 288 drops. Clearly, Bounty is better for wiping up water. This is a nice piece of logic with which the children can grapple. We also can make a nice generalization: the smaller the spot, the more water the towel can wipe up.

Question 15 should be a consumer's delight. Towels come in one-ply and two-ply. Each ply is a layer of toweling. The children are asked why some paper towels are made double in thickness. A reasonable answer should be because the water gets shared between the two layers and so doesn't spread out as much. Remember, as we just saw in Question 13, less spread means more drops picked up. To check this out, each child can pick *one* towel, fold it in half and redo the experiment. Use the worst and best towel. The spot size will have a smaller area in each case. The children then need to graph their new data. Since making good contact between the two halves is chancy, the answers will usually vary.

Question 16 is a TIMS Challenge Question that requires multiple-step reasoning. It asks which towel in Question 11 is the best buy. The children will have to understand Question 11 before they can tackle Question 16**.** The general point is that a towel with a big spot size, although not efficient in absorbing water, might be the best buy if its price were low enough. The best buy would be the towel that absorbs the most drops at the lowest price. So we need to know the number of drops absorbed by each towel (step 1) and then, given the price, determine by proportional reasoning the price per drop (step 2).

This is a hard but doable problem for bright second graders. They must know doubling and halving. Soppy costs double the generic towel (2¢ vs. 1¢), so you can buy 2 generic towels for one Soppy towel. Therefore, one Soppy towel better absorbs twice the number of drops as one generic towel. But as we saw in Question 13**,** the towel with the smaller spot area wipes up more drops. If a towel is to absorb twice the number of drops, its spot must be $\frac{1}{2}$ the size. Is the Soppy spot $\frac{1}{2}$ the area of the generic spot? Yes, indeed. In fact, since it is less than $\frac{1}{2}$, it will absorb more than double the water. So Soppy is a better buy.

Summary

This lab ranges from computing area to consumer awareness concerns. The essential element is the development of the skills used to determine area by "counting sq cm." You should emphasize this as you discuss the lab with the children. The children learn to add fractional sq cm to make whole sq cm. They also learn how symmetry can be used to make counting easier. We also probe critical thinking skills by asking the children to compare the towels' ability to absorb drops and to decide which might be a better buy.

Some aspects of this lab are relatively easy, counting sq cm, but it has some real challenges for the children. It should be fun!

Materials per Team

- 3 or 4 brands of paper towels
- eyedroppers
- water
- food coloring (optional)
- scissors
- graph paper with a sq cm grid
- containers for water