View Tube

Teacher Lab Discussion

Figure 1

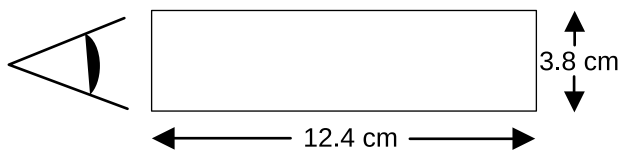
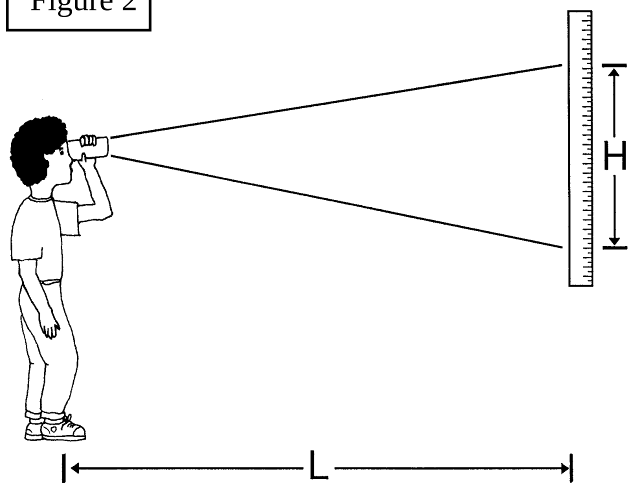


Figure 2



Overview

The *View Tube* experiment is simplicity itself. All the student has to do is look through a tube at a meterstick. Yet the experiment contains all the TIMS quantitative elements: interpolation, extrapolation, proportional reasoning, controlling variables, and inductive logic. The experiment also has marvelous applications, from learning how the Egyptians built their pyramids to measuring the size of a giraffe at the zoo. We can use the view tube as a range finder if we are lost or to measure the height of our school building.

Sticking to the maxim that cheap is best, the view tube can be the cardboard cylinder of a toilet paper roll. Our tube is illustrated in Figure 1. The dimensions will vary from brand to brand of toilet paper. That's okay.

Picture, Data Table, and Graph

For the *View Tube* experiment a meterstick is taped to the wall and the student looks at the meterstick through the tube. Depending upon where the student stands, he or she will see different lengths of the meterstick. The manipulated variable is the distance from the viewer's eye to the meterstick, L (**Question 1**), and the responding variable is the amount of the meterstick seen through the field of view, H (**Question 2**). This is illustrated in our picture of the experiment shown in Figure 2.

If you want to force the children into some subtraction, you can make them aim the view tube at the center of the ruler, and then have them subtract the numbers that are visible at the top and bottom to find H (Figure 3a). Most children like to raise the tube so that the top corresponds to the top

Figure 3

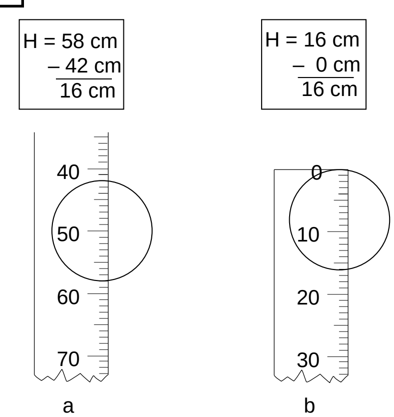
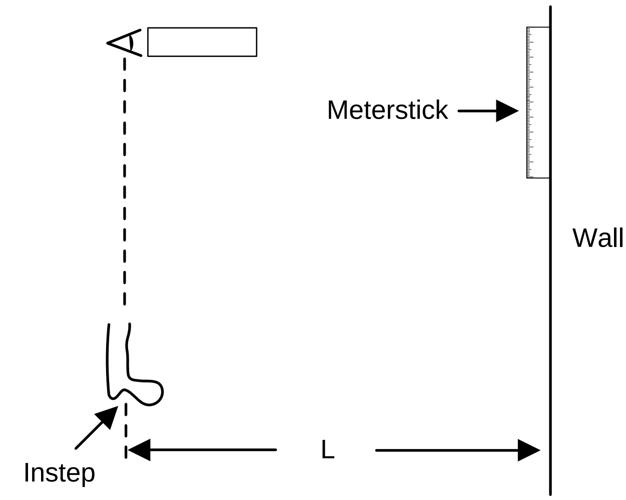


Figure 4



of the ruler and then find H by reading the number at the bottom (Figure 3b). One word of caution: the viewer must keep the tube the same distance from his or her eye for each measurement. Because of this, trading jobs is not a good idea. One partner may have glasses and the other may not; one may have deep-set eyes and the other may not. In either case they will measure different H's for the same L. On the other hand, we bring out in **Question 3** that there are fixed variables throughout the experiment. These, of course, are the *dimensions* of the view tube. Later we shall see what happens when we change those dimensions.

It is very important to measure L from the *viewer's* eye. But we do not want children poking around each other's eyes. It turns out, fortunately, that your eyes line up with the instep of your foot, as shown in Figure 4. Therefore, have the children measure L from the viewer's instep to the wall on which you have taped the meterstick.

Data from the experiment that we have carried out with our students at UIC is shown in Figure 5 and plotted in Figure 6. Because our lab at UIC is narrow, we had to pick values of L = 100 cm, 200 cm, and 300 cm, although L = 100 cm, 200 cm, and 400 cm would be preferred. In the data table of the student lab worksheets we have left the last entry under L blank for you to fill in. Stick to the latter

Figure 5

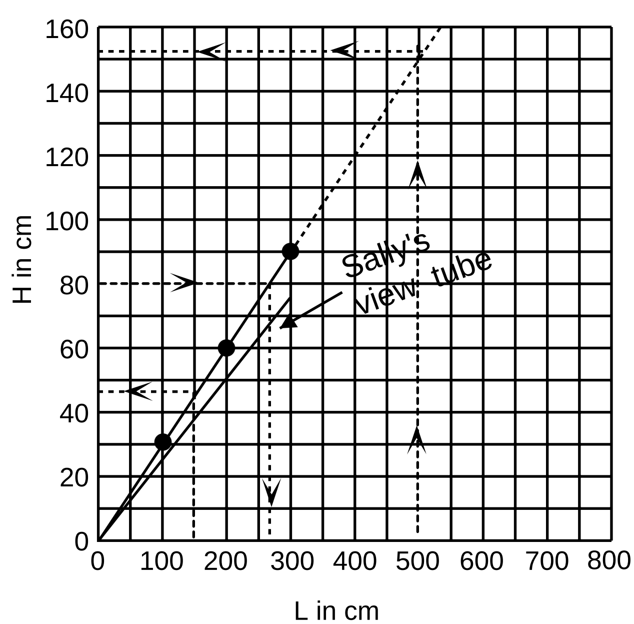
Table I

L in cm	H in cm			
	Trial 1	Trial 2	Trial 3	Average
100	32	32	32	32
200	60	64	62	62
300	90	91	91	91

set if you can. At each value of L, have the children place a small piece of masking tape on the floor. Be sure to have them remove the tape when the experiment is over.

At L = 400 cm, and even at L = 300 cm for a wide tube, H may exceed the 100 cm size of the meterstick. Great! Have the children figure out how to "stretch" their metersticks. The best way is to attach another smaller ruler or a meterstick to the top and then add the reading on the small ruler to 100 cm. But, again, let the students figure it out. The data table has room for three trials under H. Although only one child at a time does the viewing, it is still good practice if he or she does it three

Figure 6



times for two reasons. One, you want to be sure they are reading the meterstick correctly and consistently, and, two, you want them to get practice averaging. If the children are too far away to read the numbers on the meterstick, have one partner move his or her finger up and down until it comes into the field of view, and then have that partner record the values of H. This "finger" technique isn't a bad idea for all the data points. And by using both the "I see it" and "finger" technique on all the data points, the children have a double check for accuracy.

As you can see from Figure 6 the data is beautifully linear and so we are ready for questions on predicting using interpolation, extrapolation, and proportional reasoning. We shall tackle these and other ideas in the Comprehension Questions. Not surprisingly, H, the responding variable, is plotted on the vertical axis (**Question 4**), while L is plotted on the horizontal axis (**Question 5**). We ask in Question 6 if the curve is a straight line. Remember the children are fitting a curve through the data points of which (0, 0) anchors the curve. An answer to why in **Question 6** might be "Because the best fit is a straight line." A more sophisticated answer might be "Because I can see the data is proportional." And indeed we see from the data table that if we double L (from 100 cm to 200 cm), H also almost doubles (from 32 cm to 62 cm). This question is open-ended so it will be interesting to see how scientific and mathematical their answers are.

Comprehension Questions

Question 7 deals with the problem of keeping one partner as the viewer for the entire experiment. This is necessary because our eyes are not all set into our heads the same way, or one partner may wear glasses while another one does not. If your partner wears glasses, then the view tube would be farther from his or her face and he or she would see *less* of the meterstick. The children can see the effect of glasses simply by moving the view tube farther from their eyes and seeing the field of view shrink.

Questions 8 through 10 check the children's graph reading skills. The answers will depend upon the dimensions of their view tube. To make life easier on you, have the children show how they obtained their answer *right on the graph*. We do this for Questions 8, 9, and 10 in Figure 6.

In **Question 8**, we use interpolation to find out what we can see if we are150 cm from the meterstick. The answer for our view tube is approximately 47 cm. We ask the children to check their predictions and to see if they were "close." By now they should know how to answer the "close" question. They must look for the percent difference. Say they saw 50 cm, not 47 cm as predicted. There the percent difference relative to their prediction is

$$\begin{aligned}\%\text{ diff.} &= \frac{50\text{ cm} - 47\text{ cm}}{47\text{ cm}} \times 100 \\ &= .064 \times 100 \\ &= 6.4\end{aligned}$$

And this is close—well within the 10% criteria we have used before.

In **Question 9** we use extrapolation to find H when L = 5 meters, so the curve must be extended as we show in Figure 6. The answer is about H = 152 cm.

In **Question 10** we go in the opposite direction. Here the children can see 80 cm of the meterstick. How far away are they? This is what we call a range finder question. Using interpolation the answer is about 270 cm. How close was their prediction? Well, if they measure L to be 250 cm then

$$\begin{aligned}\%\text{ diff.} &= \frac{250\text{ cm} - 270\text{ cm}}{270\text{ cm}} \times 100 \\ &= -.074 \times 100 \\ &= -7.4\end{aligned}$$

Close, indeed. What does the (-) sign mean? That's right—our measurement was less than our prediction.

The answer to **Question 11** involves the dimensions of Sally's view tube which determine the curve by limiting the full view for a given length view tube. For example, if Sally were standing with the view tube right against the meterstick, then she would be close to 20 cm away and would see 5 cm. Thus she has another data point, L = 20 cm, H = 5 cm. If she plotted this point it should fall on her straight line. However, it is unrealistic to plot points whose dimensions are 15 cm or less on a graph whose scale is 100's of cm. What to do? The answer is to multiply the view tube dimensions by 10 so that we have L = 200 cm and H = 50 cm. This pair of points can easily be plotted and they should lie darn close to the curve. Thus, we can now predict what curve any view tube should give us merely by determining its dimensions. The fact that the curve is a straight line through this data point and (0,0) follows from inductive reasoning, i.e., "the curve must be a straight line for Sally's tube since it was for mine." We show Sally's curve in Figure 6.

Simple Proportional Reasoning and Applications

Because the view tube has so many applications involving proportions, it is a marvelous vehicle for studying simple proportional reasoning. We have placed five proportional reasoning questions in the Comprehension Questions, and as we answer these questions we shall discuss several of the applications of the view tube. Remember, that when solving these problems, first set up the ratio with symbols, then use the curve to determine the correct numerical value of the ratio, and finally set up the known and unknown quantities based on the question. Solving for the unknown is then straightforward, especially with calculators.

Question 12, which asks how much of the meterstick they would see if they were 6 m from it, is straightforward. Have the children convert the 6 m to 600 cm. Then we have:

$$\frac{H}{L} = \frac{62 \text{ cm}}{200 \text{ cm}} = \frac{H}{600 \text{ cm}}$$

$$\therefore \; H = 600 \text{ cm} \times \frac{62 \text{ cm}}{200 \text{ cm}} = 186 \text{ cm}$$

Notice that the units come out in cm, as they should if we properly keep track of them.

A wonderful application of the view tube is to take it to the zoo and have the children determine the sizes of the giraffes, the elephants, the snakes if they will stretch out, the bears, etc. The trick is to step back far enough to just see all of the animal in the view tube. Then, measuring L and knowing the ratio H/L for your tube you can solve for H. This is illustrated in Figure 7. Usually the animals will stand in one spot, the giraffes and elephants close to a fence or the lions and bears close to a moat, and you can measure L from that spot. **Question 13** is typical of a zoo problem. In this case you step back 18 m before you can see all of the giraffe. The 18 meters is measured by pacing and using a meterstick. Once you have L, then:

$$\frac{H}{L} = \frac{62 \text{ cm}}{200 \text{ cm}} = \frac{H}{18 \text{ m}}$$

$$\therefore \; H = 18 \text{ m} \times \frac{62 \text{ cm}}{200 \text{ cm}} = 5.6 \text{ m}$$

Figure 7

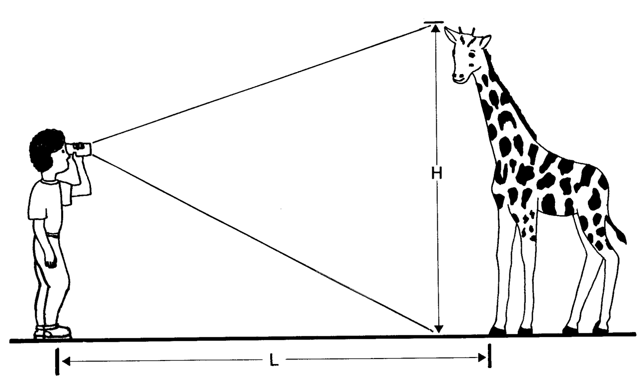
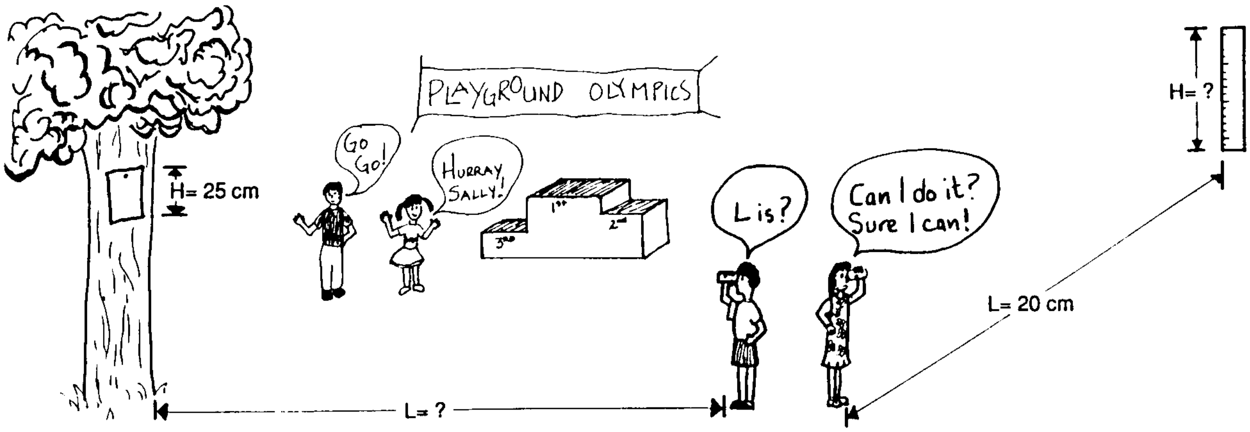


Figure 8



Notice that we have left the units of L and H in meters. This is okay because the cm units cancel out. The answer H = 5.6 meters should be converted by the children to feet since the latter units give them a bit more feel for the size of the animal and also begins to give them an idea of how big a meter really is. As an approximation (which is something we always want to do) 1 meter is about 3 feet. Therefore, our giraffe is

$$H = 5.6 \text{ m} \times \frac{3 \text{ feet}}{1 \text{ m}} = 16.8 \text{ feet}$$

The view tube makes a very effective building height measurer. As a playground exercise have the children take their tubes outside and challenge them to find the height of the school building. By backing off until they just see the entire building and by measuring L, they can find H. Or you can reverse the process and turn the view tube into a range finder. They can estimate the height of an object, say, a building from the number of stories (counting about 4 meters per story), and from this value of H they can determine L. Most of the time the building or object will not fill the view tube, but that's okay as long as you have an estimate of its height.

The idea of a range finder is brought out in **Question 14,** where Billy sees that the library only takes up $\frac{1}{3}$ of the field of view of the view tube. But Billy knows that the building is 4 stories high. Therefore, the height of the building is

$$H_{building} = 4 \text{ stories} \times \frac{4 \text{ meters}}{1 \text{ story}} = 16 \text{ meters}$$

Since the building only takes up $\frac{1}{3}$ of the view tube, the field of view covered by the view tube is

$$H_{tube} = 16 \text{ m} \times 3 = 48 \text{ m}$$

That is, at the unknown distance the view tube would just cover an object that is 48 m tall. Now we can solve for L:

(from our curve)

$$\frac{H}{L} = \frac{62 \text{ cm}}{200 \text{ cm}} = \frac{48 \text{ m}}{L}$$

$$L = 48 \text{ m} \times \frac{200 \text{ cm}}{62 \text{ cm}} = 155 \text{ meters}$$

How many blocks is 155 meters? Well, in Chicago, there are 8 blocks in a mile and a mile has 1600 meters. Therefore the length of one city block is 1600 m/8 = 200 meters. Using this as our benchmark, we find that Billy is

$$L = 155 \text{ m} \times \frac{(1 \text{ block})}{200 \text{ m}} = 0.78 \text{ blocks away}$$

We suggest that you set up an Olympic playground challenge with view tubes. Challenge the children to find out how far they are from objects whose heights are known, such as a piece of paper, or how tall a mystery stick is if they are told the distance. This is illustrated in Figure 8.

Question 15 is similar to the others that precede it, but the problem is entirely self-contained. You are told about Brian's view tube and from that you can figure out how far you are from Willie the Whale by:

$$\frac{H}{L} = \frac{25\text{ cm}}{100\text{ cm}} = \frac{15\text{ m}}{L}$$

given   find

$$\therefore L = 15\text{ m} \times \frac{100\text{ cm}}{25\text{ cm}} = 60\text{ m}$$

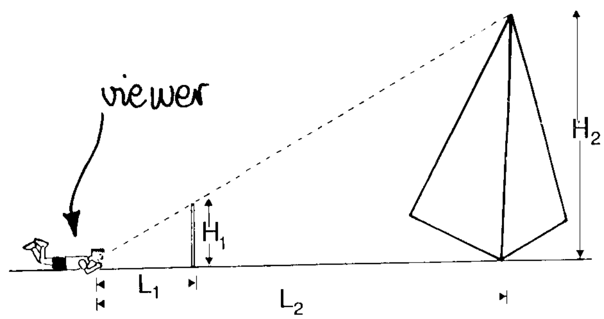
Question 16 is taken from a nationwide test administered to 13- and 17-year-olds in this country in 1979. You can see how beautifully the layout of the question fits our view tube experiment and the linearity of the physical relationship between the tops of the trees fits our linear graphical relationships for simple proportional reasoning. Amazingly, only 36% of the 13-year-olds and 50% of the 17-year-olds got the correct answer. Disgraceful! Setting it up in the TIMS manner we have:

given

$$\frac{H}{L} = \frac{6\text{ ft}}{10\text{ ft}} = \frac{H}{30\text{ ft}} \qquad \therefore H = 18\text{ ft}$$

What did the Egyptians have to do with all of this? It was probably the Egyptians who started these kinds of measurements when they tried to determine the heights of the colossal pyramids they were building. As shown in Figure 9, if a stick that is placed in the sand just blocks the top of the pyramid, then

Figure 9



$$\frac{H_1}{L_1} = \frac{H_2}{L_2}$$

If our ancient engineers walk off L_1 and L_2 and if they know H_1, then they can determine H_2, the height of the pyramid. Neat!

Finally we come to the problem of controlling and manipulating variables for the *View Tube* experiment. Besides keeping the tube always the same distance from your eye, the only other variables to control are the dimensions of the tube. If you change the dimensions of the tube, you will get a different curve. There are two dimensions to worry about, the length of the tube and its diameter. If you lengthen the tube but keep the diameter the same, you will see less. If you keep the length the same but widen the diameter, you will see more. You can bring in all kinds of tubes from wax paper rolls, from paper towels, etc., and cut them to various lengths. Between these and the toilet paper rolls you should have a variety of diameters and lengths to experiment with.

The easiest one to investigate is a tube that has the same length as yours but a different diameter. Bring in a long paper towel roll and let the kids cut it to the length of their view tube. In **Question 17,** we ask what would happen if such a tube were twice the diameter. By investigating similar situations they should find that the field of view will double. Therefore, they would see about 64 cm at 1 m, 124 cm at 2 m, and 182 cm at 3 m. In **Question 18** we double the length but keep the diameter the same. The field of view now halves from the the *initial* conditions and so you would see 16 cm at 1 m, 31 cm at 2 m, and 46 cm at 3 m.

Now the hard question, **Question 19**. We now *double* the length and *halve* the diameter. The child has to argue by deduction based on the general ideas he has just learned in the previous two questions. Doubling the length will halve the field of view, and halving the diameter will also halve the field of view. Therefore, together, the field of view will be down by a factor of 4 (that is, $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$)! The answers are, you would see 8 cm at 1 m, 15 cm at 2 m, and about 23 cm at 3 m.

Finally, in **Question 20** we generalize and show a curve for a friend's view tube. All you know is that the curve lies above yours. What can you tell about the other view tube? Based on what we have just learned, it could be shorter or it could be wider.

Summary

Incredible, isn't it, how much you can get out of a simple experiment. Doing no more than obtaining the data and plotting Figure 6, we have a nice experiment to develop length measurement skills and basic graphing and graphical analysis techniques. But then we can go on to proportional reasoning and on to the zoo, range finding, playground Olympics, and a study of ancient Egyptian building techniques. All for one toilet paper roll and a meterstick. This is what elementary school science should be all about.

Materials per Team

- 1 view tube (eg., cardboard tube from a toilet paper roll)
- 1 or 2 metersticks
- 1 12" ruler