Taste of TIMS

Teacher Lab Discussion

Overview

Several years ago, we were trying to develop mass experiments for the 3rd, 4th, and 5th grades. One day one of our teachers came to a Saturday class and said that she had her 4th graders mass the apples they were eating during a rest period. They would take a bite, mass it, take another, mass it, etc. The children had so much fun that almost all of them stayed in at lunch (on a nice May day) and did the same for their sandwiches and even cookies. Hence was born *Taste of TIMS.*

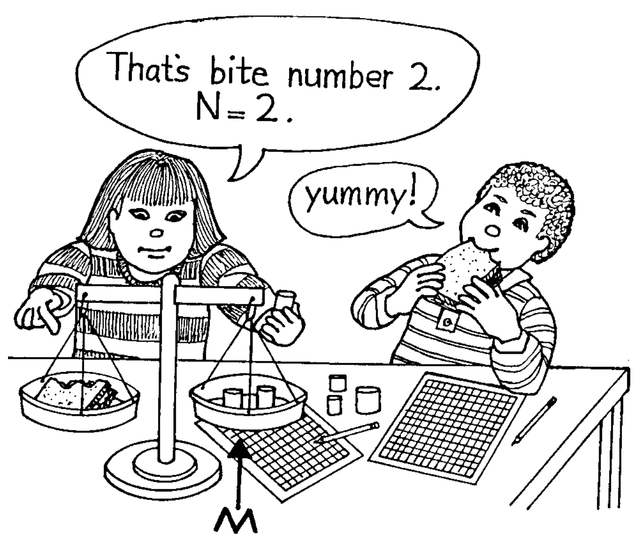
This experiment will stick to sandwiches. The children should bring one sandwich ready for eating. The type of bread or filling is not important. Each partner eats his or her own sandwich. The sandwich must be eaten whole, not sliced in two, no matter what etiquette dictates. There should be two slices of bread. Tell the children to bring in enough information so they can calculate the mass of each slice. By investigating the package the bread comes in, they should find the total number of slices and the mass of the loaf. We shall work on the calculation in the comprehension question section. They will not mass the bread slices separately, only the entire sandwich.

Although the experiment may seem farfetched, it really isn't. Often scientists deal with objects that are "disappearing" and so they have to make measurements about the object while it is in a state of flux. Liquids evaporate, animal populations decline, plants and animals lose mass, stock market prices fall. In all of these situations, we start with an initial amount of something and go downhill—so to speak—from there. This will be the second time the children have seen a negative slope. The first was in *Evaporation I.* We shall follow up this idea in grade five with *Candle Burning III* and again in grade 6 with *Evaporation II* and *Hung Out to Dry.* In studying *Taste of TIMS*, we shall analyze the situation using concrete operational skills. Later in grade 5 and in grade 6, we will deal with this negative slope problem using formal operational techniques.

Picture, Data Table, and Graph

The picture should show what the children are doing, how they do it, and identify the manipulated and responding variables. Our picture is shown in Figure 1. The number of bites, N, is the manipulative variable (**Question 1**), and the remaining mass, M, is the responding variable (**Question 2**). Although we could calculate the mass of each bite, the raw data is what remains of the sandwich and not what is down one's stomach.

Figure 1



One cannot very well get at the latter to make a direct measurement. In **Question 3**, we ask what variable or variables are controlled during the experiment. The main point we are after here is that each bite should remain as close to the same size as possible. This is why we do not slice the sandwich. We want to prolong the last bite as long as possible so we do not have a fraction of a bite left over unil the very end.

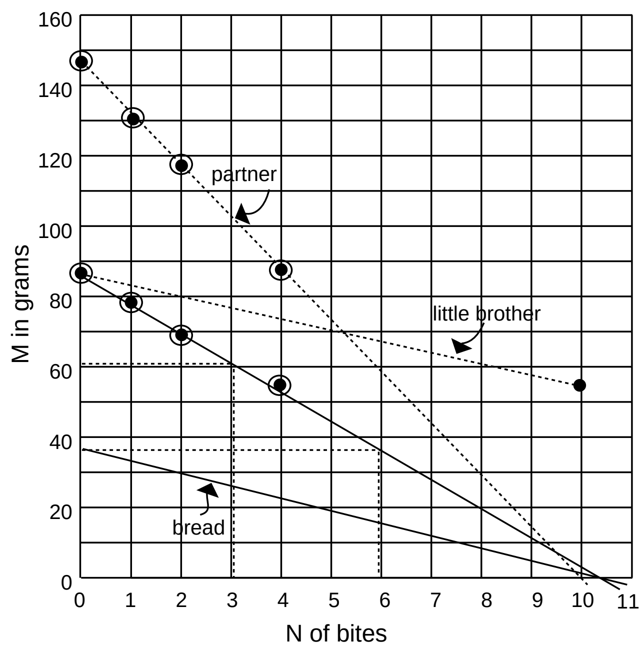
We instruct the children to collect the data taking 1, 2, and 4 bites. We take a limited number of set bites so that we can ask interpolation questions (N = 3) and extrapolation questions. For the latter, we want the children to be able to check their prediction and that will hardly be possible if the data has been eaten! We have set the table up for the 1-gm, 5-gm, and 8-gm TIMS standard masses. The results are shown in Figure 2 for a salami sandwich. The children then put the sandwich aside for a few minutes and plot the data. We want to get through the curve and the first three comprehension questions before they finish the sandwich, so start the data taking at the beginning of lunch and let lunch go on for about $1\frac{1}{2}$ hrs.

Before the children fit a curve to the data, we ask in **Question 4** what data point must the curve pass through. The answer is the only error free point that we have, N = 0, M = 87 grams. For all the rest it is hard to control the bite size and so we expect these may not lie on the curve. We show the data

Figure 2

N of bites	M in grams			
	1 gm	5 gm	8 gm	Total
0	2	1	10	87
1	2	1	9	79
2	1	1	8	70
4	2	1	6	55

Figure 3



plotted in Figure 3 and our fit to the four data points. Actually ours look pretty good with only the N = 4 point to be a little bit above our fit. But no matter, the fit must go through the N = 0 data point.

Now we are ready to answer a few comprehension questions and then finish lunch.

Comprehension Questions

In **Question 5**, we ask the children to use their graph to predict the mass of the sandwich after their third bite. Even though the curve does not pass through (0,0), the children can still use interpolation to answer the question. As shown in Figure 3, the prediction for our data is M(3) = 61 grams. We devilishly ask if they can check their prediction. The answer is no because, “we have already swallowed the third bite.” In **Question 6**, because the curve is a straight line, they can easily use extrapolation to predict the mass after 6 bites. Again, using our graph, as shown in Figure 3, the prediction is about

37 grams. Now they can check their prediction by taking two more bites. For our raw data M(6) = 36 grams.

By now, the children should be starving, so after they predict, in **Question 7**, how many bites it would take to eat their entire sandwich, they can check their prediction by finishing lunch. The prediction is again straightforward. All they have to do is extend the straight line until it intersects the horizontal axis where M is equal to zero. We can see that this occurs for N between 10 and 11 bites. That means that after 10 bites there is still a bit left, so the eleventh bite does it, the sandwich is gone. And, indeed, it took a total of eleven bites for us to finish our sandwich.

Question 8 is an open-ended question where the children are asked how close their prediction was in Question 6. This is a question that gives you an opportunity to see just what they have learned so far about these kinds of comparisons. What they should not do is take the difference between predicted and measured as the sole measure of comparisons. In this case the difference is 37 grams – 36 grams = 1 gram. But there is nothing to compare with the 1 gram. But they should know by now that they need to find the percentage difference, say, with respect to the measured value. That is:

$$\%\text{ diff.} = \frac{M_{pred} - M_{meas}}{M_{meas}} = \frac{37\text{ gm} - 36\text{ gm}}{36\text{ gms}} = \frac{1\text{ gm}}{36\text{ gm}} = \frac{.028\text{ gm}}{1\text{ gm}}$$

which is 2.8 grams in 100 grams or 2.8 percent. And 2.8% is close.

Question 9 is harder. We ask, "on the average, how much mass do you swallow each bite?" In a sneaky way, we are asking what the magnitude of the slope of the curve is without really telling the children. What they should not do is find one bite, say the first, and say the mass swallowed is 87 gm – 79 gm = 8 gm. In the second bite, the mass swallowed is 79 gm – 70 gm = 9 gm, and so on. So we do need an average. In a sense we get that by drawing the straight line fit to the data. The line is our average fit to the data so it tells us what the average mass swallowed is per bite. Although one can find this from any one bite, it is better to take a large number of bites, say 9, and find Δ M(9) and then divide by 9 bites. Again, using our curve Δ M(9) = 87gms – 15gms = 72gms. The average would be

$$\begin{pmatrix}\text{Average swallowed}\\ \text{per bite}\end{pmatrix} = \frac{\Delta M}{\Delta N} = \frac{87\text{ gm} - 15\text{ gm}}{9\text{ bites}} = \frac{72\text{ gm}}{9\text{ bites}}$$

$$\text{Therefore,}\quad \frac{72\text{ gm}}{9\text{ bites}} = \frac{\Delta M}{1\text{ bite}}$$

$$\text{Solving,}\quad \Delta M = \frac{72\text{ gm}}{9\text{ bites}} \times 1\text{ bite} = 8\text{ gm}$$

In **Question 10** we ask them to plot their partner's data on their curve. Our partner was Andy and his data is plotted in Figure 3. We ask in what way the partner's data is different and the same. The differences are quite striking: (1) Andy's initial mass M(0) is much bigger; (2) the slopes are different, that is, the average mass swallowed is bigger; (3) the number of bites to finish the sandwich is smaller for Andy, 10 vs. 11 bites. Encourage the children to be as quantitative as possible when answering this part of the question. But there is also a striking similarity. Both curves are straight lines and this is an important inductive generalization they should draw from the data.

Tongue in cheek, we ask in **Question 11a** who has the biggest mouth. For our example, Andy wins "mouths down," so to speak. A class mouth off is necessary to answer **Question 11b**. Teachers can not enter. Remember it is the slope that counts and not the mass of the original sandwich or the number of bites to finish it off.

In **Question 12** we move in the other direction, looking at a smaller-sized bite. Our fictitious brother only averages three grams per bite. The children are asked to draw his curve on their graph. How should they go about doing it? Give them a chance to come up with their own solutions. One is to subtract 3 grams from 87 grams and plot 84 grams at N = 1 and then connect the two data points. To reduce the error in connecting two numbers so close together, it would be better if the children would take ten bites and plot 87 gm – 30 gm = 57gm at N = 10 bites. Then they draw in the line. That is what we have done in Figure 3.

In **Question 13** we ask how many bites it would take the brother to finish your sandwich. This is a hard problem. The concrete operational way to solve the problem is to extrapolate to M = 0 grams, but we doubt if there is enough room to do this. So the children will have to use a formal operational approach. We would suggest all the children be asked to solve the problem analytically. One way to do this is to use the graph to find out how many bites it takes to finish $\frac{1}{4}$ of the sandwich and then multiply that number by four. Now $\frac{1}{4}$ of 87 is 87 gm/4 = 21.75 gm or 22 grams. That means you have reduced the sandwich to a mass of 87 gm – 22 gm = 65 grams. Reading N off the curve for that value of M, we see that it takes 7 bites to reduce the mass to 65grams. Multiply that by 4 and we have the number of bites for the brother to polish off the sandwich, $N = 7 \times 4 = 28$. This is multiple logic and is not easy. A rather more clever and insightful way to solve the problem is to notice that little brother has reduced the sandwich by 3 grams in 1 bite. Therefore, one can set up a ratio.

$$\frac{\Delta M}{N} = \frac{3\text{ gm}}{1\text{ bite}} = \frac{87\text{ gm}}{N}$$

$$\text{Solving for N,} \qquad N = \frac{87\text{ gm} \times 1\text{ bite}}{3\text{ gm}} = 29\text{ bites}$$

This would be a good homework problem which you can then open to class discussion as the children explore different approaches to solving it.

In **Question 14** the children determine how much of their sandwich is bread and how much filling. As mentioned in the overview, they can find the necessary information on their package of bread. Our package contained 23 slices and had a total mass of 454 grams (1pound). Thus,

$$\frac{M}{S} = \frac{454\text{ gm}}{23\text{ slices}} = \frac{M}{1\text{ slice}}$$

Therefore, the mass of one slice of bread is

$$M = \frac{454\text{ gm}}{23\text{ slices}} \times 1\text{ slice} \cong 20\text{ gm}$$

Since we have two slices, M bread = 40 grams. And the filling is, therefore,

$$\begin{aligned} M_{filling} &= M_{sand} - M_{bread} \\ &= 87\text{ gm} - 40\text{ gm} \\ &= 47\text{ gm} \end{aligned}$$

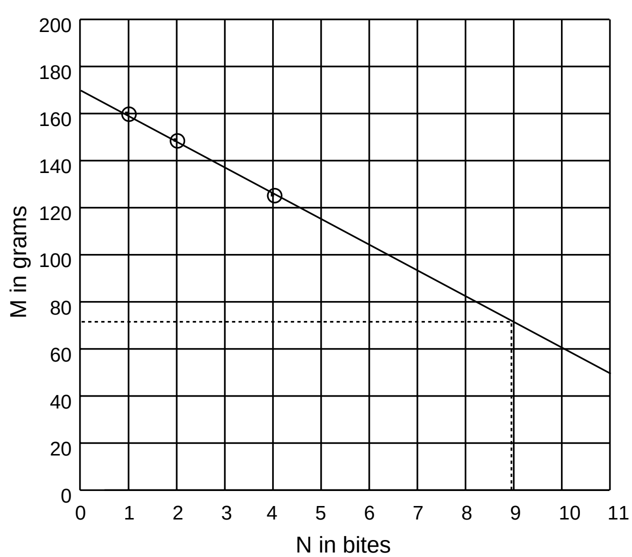
In **Question 15** the children are asked to find the curve of M vs. N for bread alone. It is easy to draw if they will think a moment. The curve starts at N = 0 and M = 40 grams and ends at M = 0 grams and

N at exactly where their original curve crosses the horizontal axis. This is shown in Figure 3.

Questions 16, 17, and 18 can be homework problems or exam questions or a combination. In **Question 16** we qualitatively explore the two key points about these types of curves. Graphs for 3 pairs of partners are shown. We ask what is the same and different about each of the three sets of graphs. For partners A they both start with the same mass apples but the slopes of the curves, that is, the bite sizes, are different. For B the apples have different masses but the partners have the same bite sizes. For C nothing is the same. Both the initial mass of the apples and the bite sizes are different.

In **Question 17** the children repeat the experiment with fixed data. We give them the grid, they do all the rest including labeling the axis, finding the right scale for the axis, and plotting the data. The graph should look like the one in Figure 4. We ask what the mass of the apple would be after 9 bites. Using extrapolation, the answer is about 75 grams as shown in Figure 4. How many bites would it take to eat the entire apple, core and all? Recalling our discussion in Question 13, the children can try several different approaches. Using a half apple technique, one can read off the graph the number of bites to reduce the apple to 168/2 = 84grams. Reading N off the curve, we get N = 7.75 bites for M = 84 grams. Doubling that gives a total of 15.5 bites which translates into 16 real bites to eat the whole apple.

Figure 4



In **Question 18** we ask what is the average mass of the apple bitten off in one bite. Using the above data

$$\frac{\Delta M}{\Delta N} = \frac{165\text{ gm}}{15.5\text{ bites}} = \frac{\Delta M}{1\text{ bite}}$$

Solving,

$$\Delta M = 10.57\text{ gm per bite}$$

We conclude in **Question 19** with a TIMS challenge question: Are the variables N and M proportional in these experiments? Since they are in the middle of the 5th grade, we hope the children can rise to the challenge of the question and give you a convincing answer, which is NO, they are not proportional. Why? Because the ratio of M/N is not constant. For example, in Figure 4, when M = 160 gm, N = 1, so the ratio is

$$\frac{M}{N} = \frac{160\text{ gm}}{1\text{ bite}}$$

But when M = 100 gm, N = 6.5 bites.

Therefore,

$$\frac{M}{N} = \frac{100\text{ gm}}{6.5} = 15.4\text{ gm}$$

This is a much smaller ratio than the first and, indeed, with M falling and N getting bigger, the ratio must go to zero when M goes to zero. Yes, a nice challenge question.

Summary

In *Taste of TIMS* we introduce the children to disappearing data and how to analyze it. This is another in our line of experiments where the data does not pass through (0,0). Previously the slope has been up; now we look at a downward slope. But the curve is still a straight line and so we can use interpolation and most especially extrapolation, but not simple proportional reasoning to solve problems.

We challenge the children to measure mass using the equal arm balance and standard masses and to predict a variety of things about their vanishing lunch. Both concrete and formal operational logic is used.

Carrying out the experiment has proved to be a fun activity. The real challenge, however, is using a variety of analytical skills to analyze the results.

Materials per Team

- one equal arm balance
- set of standard TIMS masses
- one sandwich per partner—unsliced